Practical Information on Gears

This chapter provides fundamental theoretical and practical information about gearing. It also introduces various gear-related standards as an aid for the designer who is going to use gears for his planning.

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1 GEAR TOOTH MODIFICATIONS

Intentional deviations from the involute tooth profile are used to avoid excessive tooth load deflection interference and thereby enhances load capacity. Also, the elimination of tip interference reduces meshing noise. Other modifications can accommodate assembly misalignment and thus preserve load capacity.

(1) Tooth Tip Relief

There are two types of tooth relief. One modifies the addendum, and the other the dedendum. See Figure 1.1. Tip relief is much more popular than root modification.
Care should be taken, however, not to modify excessively since that will cause bad effect in meshing.

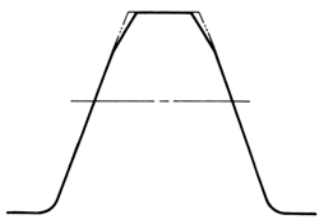


Fig. 1.1 Tip relief

(2) Crowning and End Relief

Crowning and end relief are tooth surface modifications in the axial direction.
Crowning is the removal of a slight amount of tooth from the center on out to reach edge, making the tooth surface slightly convex. This method allows the gear to maintain contact in the central region of the tooth and permits avoidance of edge contact with consequent lower load capacity. Crowning also allows a greater tolerance in the misalignment of gears in their assembly, maintaining central contact. The crowning should not be larger than necessary as otherwise it would reduce dimentions of tooth contact, thus weakening durable strength.
End relief is the chamfering of both ends of tooth surface. See Figure 1.2.

Fig. 1.2 Crowning and end relief

(3) Topping And Semitopping

In topping, often referred to as top hobbing, the top or tip diameter of the gear is cut simultaneously with the generation of the teeth. See page 387 "The Generating of a Spur Gear". Also, refer to Figure 3.5, 3.6 and 3.7 in that section. An advantage is that there will be no burrs on the tooth top. Also, the tip diameter is highly concentric with the pitch circle. Semitopping is the chamfering of the tooth's top corner, which is accomplished simultaneously with tooth generation. Figure 1.3 shows a semitopping cutter and the resultant generated semitopped gear. Such a tooth tends to prevent corner damage. Also, it has no burr. The magnitude of semitopping should not go beyond a proper limit as otherwise it would significantly shorten the addendum and contact ratio.

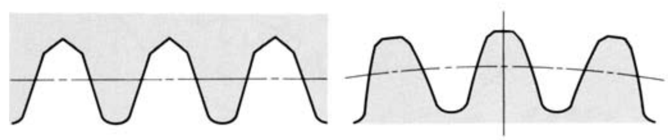


Fig.1.3 Semitopping cutter and the gear profile generated

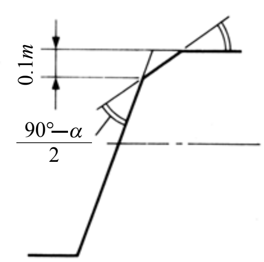


Fig.1.4 Recommended magnitude of semitopping

2 GEAR TRAINS

2.1 Planetary Gear System

The basic form of a planetary gear system is shown in Figure 2.1. It consists of a sun gear A, planet gears B, internal gear C and carrier D.

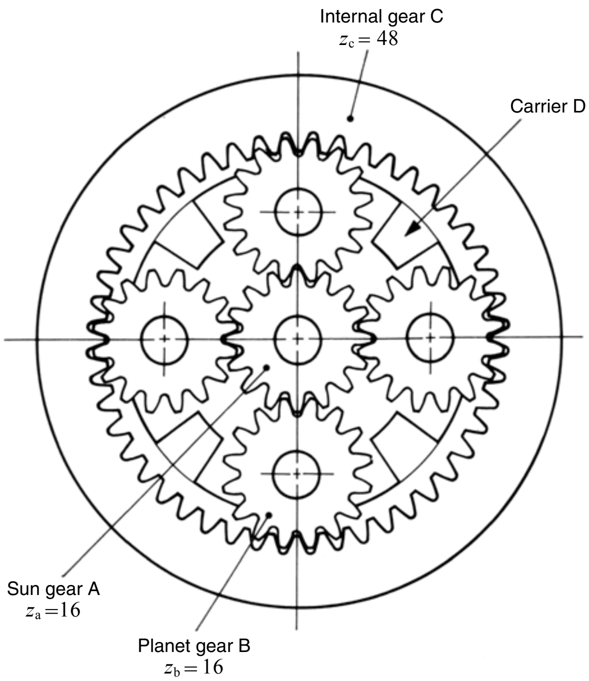


Fig.2.1 An example of a planetary gear system

The input and output axes of a planetary gear system are on a same line. Usually, it uses two or more planet gears to balance the load evenly. It is compact in space, but complex in structure. Planetary gear systems need a high-quality manufacturing process. The load division between planet gears, the interference of the internal gear, the balance and vibration of the rotating carrier, and the hazard of jamming, etc. are inherent problems to be solved.
Figure 2.1 is a so called 2K-H type planetary gear system. The sun gear, internal gear, and the carrier have a common axis.

(1) Relationship Among the Gears in a Planetary Gear System
In order to determine the relationship among the numbers of teeth of the sun gear A (z_a), the planet gears B (z_b) and the internal gear C (z_c) and the number of planet gears (N) in the system, the parameters must satisfy the following three conditions:

Condition No.1:
$$z_c = z_a + 2z_b \tag{2.1}$$

This is the condition necessary for the center distances of the gears to match. Since the equation is true only for the standard gear system, it is possible to vary the numbers of teeth by using profile shifted gear designs.

To use profile shifted gears, it is necessary to match the center distance between the sun A and planet B gears, a_1, and the center distance between the planet B and internal C gears, a_2.

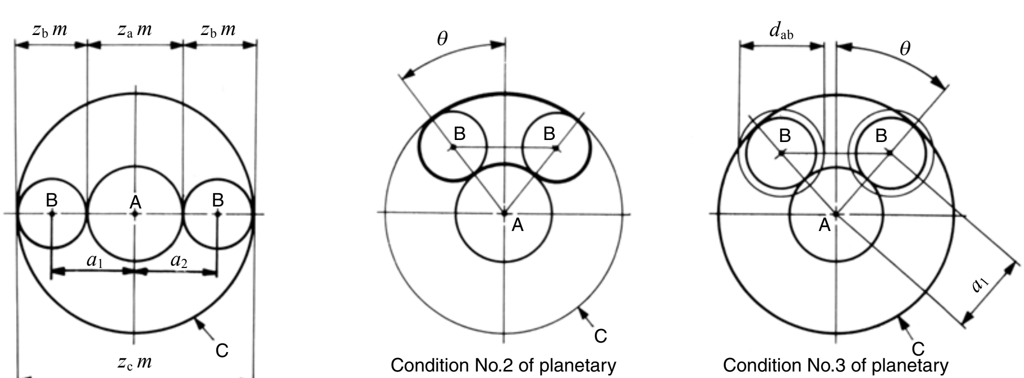
$$a_1 = a_2 \tag{2.2}$$

Condition No.2:
$$\frac{z_a + z_c}{N} = \text{Integer} \tag{2.3}$$

This is the condition necessary for placing planet gears evenly spaced around the sun gear. If an uneven placement of planet gears is desired, then Equation (2.4) must be satisfied.

$$\frac{(z_a + z_c)\theta}{180} = \text{Integer} \tag{2.4}$$

Where θ : half the angle between adjacent planet gears (°)



Condition No.1 of planetary gear system

Condition No.2 of planetary gear system

Condition No.3 of planetary gear system

Fig.2.2 Conditions for selecting gears

Condition No.3 $\quad z_b+2 < (z_a+z_b)\sin\dfrac{180°}{N} \qquad (2.5)$

Satisfying this condition insures that adjacent planet gears can operate without interfering with each other. This is the condition that must be met for standard gear design with equal placement of planet gears. For other conditions, the system must satisfy the relationship:

$$d_{ab} < 2a_1 \sin\theta \qquad (2.6)$$

Where:

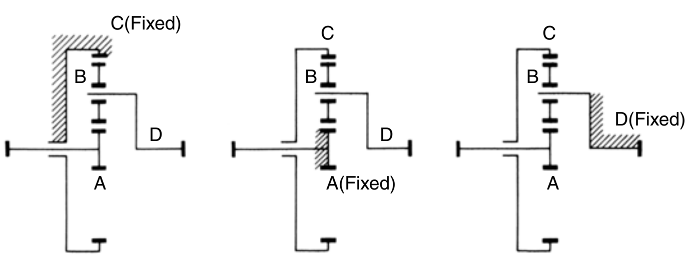
d_{ab} : tip diameter of the planet gears

a_1 : center distance between the sun and planet gears

Besides the above three basic conditions, there can be an interference problem between the internal gear C and the planet gears B. See Section 4.2 Internal Gears (Page 394).

(2) Transmission Radio of Planetary Gear System

In a planetary gear system, the transmission ratio and the direction of rotation would be changed according to which member is fixed. Figure 2.3 contain three typical types of planetary gear mechanisms,



(a) Planetary type (b) Solar type (c) Star type

Fig.2.3 Planetary gear mechanism

(a) Planetary Type

In this type, the internal gear is fixed. The input is the sun gear and the output is carrier D. The transmission ratio is calculated as in Table 2.1.

Table 2.1 Equations of transmission ratio for a planetary type

No.	Description	Sun gear A z_a	Planet gear B z_b	Internal gearC z_c	Carrier D
1	Rotate sun gear a once while holding carrier	$+1$	$-\frac{z_a}{z_b}$	$-\frac{z_a}{z_c}$	0
2	System is fixed as a whole while rotating	$+\frac{z_a}{z_c}$	$+\frac{z_a}{z_c}$	$+\frac{z_a}{z_c}$	$+\frac{z_a}{z_c}$
3	Sum of 1 and 2	$1+\frac{z_a}{z_c}$	$\frac{z_a}{z_c}-\frac{z_a}{z_b}$	0 (fixed)	$+\frac{z_a}{z_c}$

$$\text{Transmission ratio} = \frac{1+\dfrac{z_a}{z_c}}{\dfrac{z_a}{z_c}} = \frac{\dfrac{z_c}{z_a}+1}{1} \qquad (2.7)$$

Note that the direction of rotation of input and output axes are the same. Example: $z_a = 16$, $z_b = 16$, $z_c = 48$, then transmission ratio = 4.

(b) Solar Type

In this type, the sun gear is fixed. The internal gear C is the input, and carrier D axis is the output. The speed ratio is calculated as in Table 2.2.

Table 2.2 Equations of transmission ratio for a solar type

No.	Description	Sun gearA z_a	Planet gearB z_b	Internal gearC z_c	Carrier D
1	Rotate sun gear a once while holding carrier	$+1$	$-\frac{z_a}{z_b}$	$-\frac{z_a}{z_c}$	0
2	System is fixed as a whole while rotating	-1	-1	-1	-1
3	Sum of 1 and 2	0 (fixed)	$-\frac{z_a}{z_b}-1$	$-\frac{z_a}{z_c}-1$	-1

$$\text{Transmission ratio} = \frac{-\dfrac{z_a}{z_c}-1}{-1} = \frac{\dfrac{z_a}{z_c}+1}{1} \qquad (2.8)$$

Note that the directions of rotation of input and output axes are the same.
Example: $z_a = 16$, $z_b = 16$, $z_c = 48$,
then the transmission ratio = 1.3333333

(c) Star Type

This is the type in which Carrier D is fixed. The planet gears B rotate only on fixed axes. In a strict definition, this train loses the features of a planetray system and it becomes an ordinary gear train. The sun gear is an input axis and the internal gear is the output. The transmission ratio is :

$$\text{Transmission Ratio} = -\frac{z_c}{z_a} \qquad (2.9)$$

Referring to Figure 2.3(c), the planet gears are merely idlers. Input and output axes have opposite rotations.
Example: $z_a = 16$, $z_b = 16$, $z_c = 48$;
then transmission ratio = -3 .

2.2 Hypocycloid Mechanism

In the meshing of an internal gear and an external gear, if the difference in numbers of teeth of two gears is quite small, a profile shifted gear could prevent the interference. Table 2.3 is an example of how to prevent interference under the conditions of z_2 = 50 and the difference of numbers of teeth of two gears ranges from 1 to 8.

Table 2.3 The meshing of internal and external gears of small difference of numbers of teeth $m = 1,\ \alpha = 20°$

z_1	49	48	47	46	45	44	43	42
x_1	0							
z_2	50							
x_2	1.00	0.60	0.40	0.30	0.20	0.11	0.06	0.01
α_b	61.0605°	46.0324°	37.4155°	32.4521°	28.2019°	24.5356°	22.3755°	20.3854°
a	0.971	1.354	1.775	2.227	2.666	3.099	3.557	4.010
ε	1.105	1.512	1.726	1.835	1.933	2.014	2.053	2.088

All combinations above will not cause involute interference or trochoid interference, but trimming interference is still there. In order to assemble successfully, the external gear should be assembled by inserting in the axial direction. A profile shifted internal gear and external gear, in which the difference of numbers of teeth is small, belong to the field of hypocyclic mechanism, which can produce a large reduction ratio in single step, such as 1/100.

$$\text{Transmission ratio} = \frac{z_1}{z_2 - z_1} \qquad (2.10)$$

In Figure 2.4 the gear train has a difference of numbers of teeth of only 1; z_1 = 30 and z_2 = 31. This results in a transmission ratio of 30.

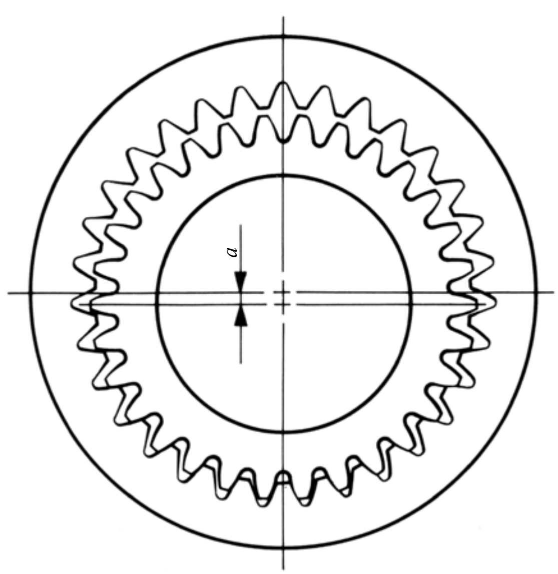


Fig.2.4 The meshing of internal gear and external gear in which the numbers of teeth difference is 1

2.3 Constrained Gear System

A planetary gear system which has four gears is an example of a constrained gear system. It is a closed loop system in which the power is transmitted from the driving gear through other gears and eventually to the driven gear. A closed loop gear system will not work if the gears do not meet specific conditions.

Let z_1, z_2 and z_3 be the numbers of gear teeth, as in Figure 2.5. Meshing cannot function if the length of the heavy line (belt) does not divide evenly by pitch. Equation (2.11) defines this condition.

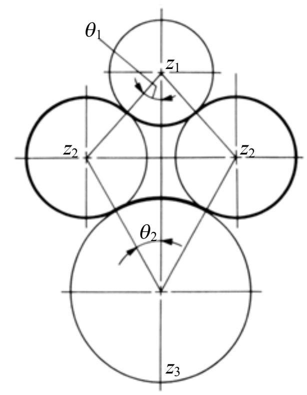


Fig.2.5 Constrained gear system

$$\frac{z_1\theta_1}{180} + \frac{z_2\,(180 + \theta_1 + \theta_2)}{180} + \frac{z_3\theta_2}{180} = \text{integer} \qquad (2.11)$$

Figure 2.6 shows a constrained gear system in which a rack is meshed. The heavy line in Figure 2.6 corresponds to the belt in Figure 2.5. If the length of the belt cannot be evenly divided by pitch then the system does not work. It is described by Equation (2.12).

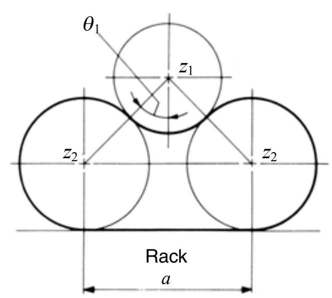


Fig.2.6 Constrained gear system containing a rack

$$\frac{z_1\theta_1}{180} + \frac{z_a\,(180 + \theta_1)}{180} + \frac{a}{\pi m} = \text{integer} \qquad (2.12)$$

3 TOOTH THICKNESS

There are direct and indirect methods for measuring tooth thickness. In general, there are three methods:

- Chordal tooth thickness measurement
- Span measurement
- Over pin or ball measurement

3.1 Chordal Tooth Thickness Measurement

This method employs a tooth caliper that is referenced from the gear's tip diameter. Thickness is measured at the reference circle. See Figure 3.1.

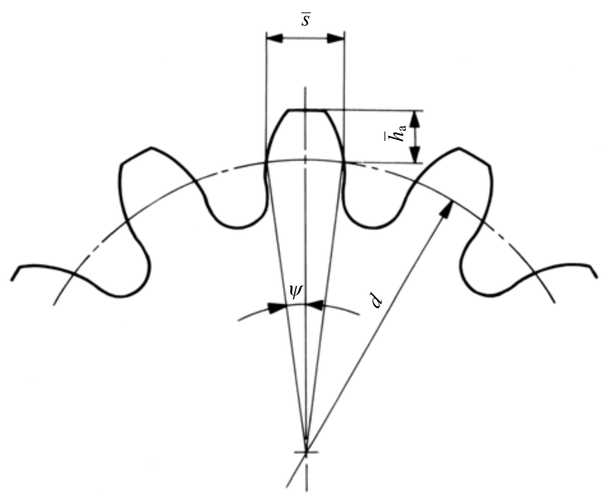


Fig.3.1 Chordal tooth thickness method

(1) Spur Gears

Table 3.1 presents equations for each chordal tooth thickness measurement.

Table 3.1 Equations for spur gear chordal tooth thickness

No.	Item	Symbol	Formula	Exmple
1	Tooth thickness	s	$\left(\frac{\pi}{2} + 2x\tan\alpha\right)m$	m =10 α =20° z =12 x = + 0.3 h_a =13.000 s =17.8918 ψ = 8.54270° $\bar{s}$ =17.8256 $\bar{h}_a$ =13.6657
2	Tooth thickness half angle	ψ	$\frac{90}{z} + \frac{360x\tan\alpha}{\pi z}$	
3	Chordal tooth thickness	$\bar{s}$	$zm\sin\psi$	
4	Chordal height	$\bar{h}_a$	$\frac{zm}{2}(1 - \cos\psi) + h_a$	

(2) Spur Racks and Helical Racks

The governing equations become simple since the rack tooth profile is trapezoid, as shown in Table 3.2.

Table 3.2 Chordal tooth thickness of racks

No.	Item	Symbol	Formula	Example
1	Chordal tooth thickness	$\bar{s}$	$\frac{\pi m}{2}$ or $\frac{\pi m_n}{2}$	m = 3 α = 20° $\bar{s}$ = 4.7124 $\bar{h}_a$ = 3.0000
2	Chordal height	$\bar{h}_a$	h_a	

NOTE: These equations are also applicable to helical racks.

(3) Helical Gears

The chordal tooth thickness of helical gears should be measured on the normal plane basis as shown in Table 3.3. Table 3.4 presents the equations for chordal tooth thickness of helical gears in the transverse system.

Table 3.3 Equations for chordal tooth thickness of helical gears in the normal system

No.	Item	Symbol	Formula	Example
1	Normal tooth thickness	s_n	$\left(\frac{\pi}{2} + 2x_n \tan\alpha_n\right) m_n$	$m_n = 5$ $\alpha_n = 20°$ $\beta = 25°\ 00'\ 00''$ $z = 16$ $x_n = +0.2$ $h_a = 6.0000$ $s = 8.5819$ $z_v = 21.4928$ $\psi_v = 4.57556°$ $\bar{s} = 8.5728$ $\bar{h}_a = 6.1712$
2	Number of teeth of an equivalent spur gear	z_v	$\frac{z}{\cos^3\beta}$	
3	Tooth thickness half angle	ψ_v	$\frac{90}{z_v} + \frac{360\, x_n \tan\alpha_n}{\pi z_v}$	
4	Chordal tooth thickness	$\bar{s}$	$z_v\, m_n \sin\psi_v$	
5	Chordal height	$\bar{h}_a$	$\frac{z_v\, m_n}{2}(1 - \cos\psi_v) + h_a$	

Table 3.4 Equations for chordal tooth thickness of helical gears in the transverse system

No.	Item	Symbol	Formula	Example
1	Normal tooth thickness	s_n	$\left(\frac{\pi}{2} + 2x_t \tan\alpha_t\right) m_t \cos\beta$	$m_t = 4$ $\alpha_t = 20°$ $\beta = 22°\ 30'\ 00''$ $z = 20$ $x_t = +0.3$ $h_a = 4.7184$ $s = 6.6119$ $z_v = 25.3620$ $\psi_v = 4.04196°$ $\bar{s} = 6.6065$ $\bar{h}_a = 4.8350$
2	Numer of teeth in an equivalent spur gear	z_v	$\frac{z}{\cos^3\beta}$	
3	Tooth thickness half angle	ψ_v	$\frac{90}{z_v} + \frac{360 x_t \tan\alpha_t}{\pi z_v}$	
4	Chordal tooth thickness	$\bar{s}$	$z_v\, m_t \cos\beta\ \sin\ \psi_v$	
5	Chordal height	$\bar{h}_a$	$\frac{z_v\, m_t \cos\beta}{2}(1 - \cos\psi_v) + h_a$	

NOTE: Table 3.4 equations are also for the tooth profile of a Sunderland gear.

(4) Bevel Gears

Table 3.5 shows the the equations for chordal tooth thickness of a Gleason straight bevel gear. Table 3.6 shows the same of a standard straight bevel gear. And Table 3.7 the same of a Gleason spiral bevel gear.

Table 3.5 Equations for chordal tooth thickness of gleason straight bevel gears

No.	Item	Symbol	Formula	Example
1	Tooth thickness factor (Coefficient of horizontal profile shift)	K	Obtain from Figure 3.2	$m = 4$ $\alpha = 20°$ $\Sigma = 90°$ $z_1 = 16$ $z_2 = 40$ $z_1/z_2 = 0.4$ $K = 0.0259$ $h_{a1} = 5.5456$ $h_{a2} = 2.4544$ $\delta_1 = 21.8014°$ $\delta_2 = 68.1986°$ $s_1 = 7.5119$ $s_2 = 5.0545$ $\bar{s}_1 = 7.4946$ $\bar{s}_2 = 5.0536$ $\bar{h}_{a1} = 5.7502$ $\bar{h}_{a2} = 2.4692$
2	Tooth thickness	s_1 s_2	$\pi m - s_2$ $\frac{\pi m}{2} - (h_{a1} - h_{a2})\tan\alpha - Km$	
3	Chordal tooth thickness	$\bar{s}$	$s - \frac{s^3}{6\,d^2}$	
4	Chordal height	$\bar{h}_a$	$h_a + \frac{s^2\cos\delta}{4d}$	

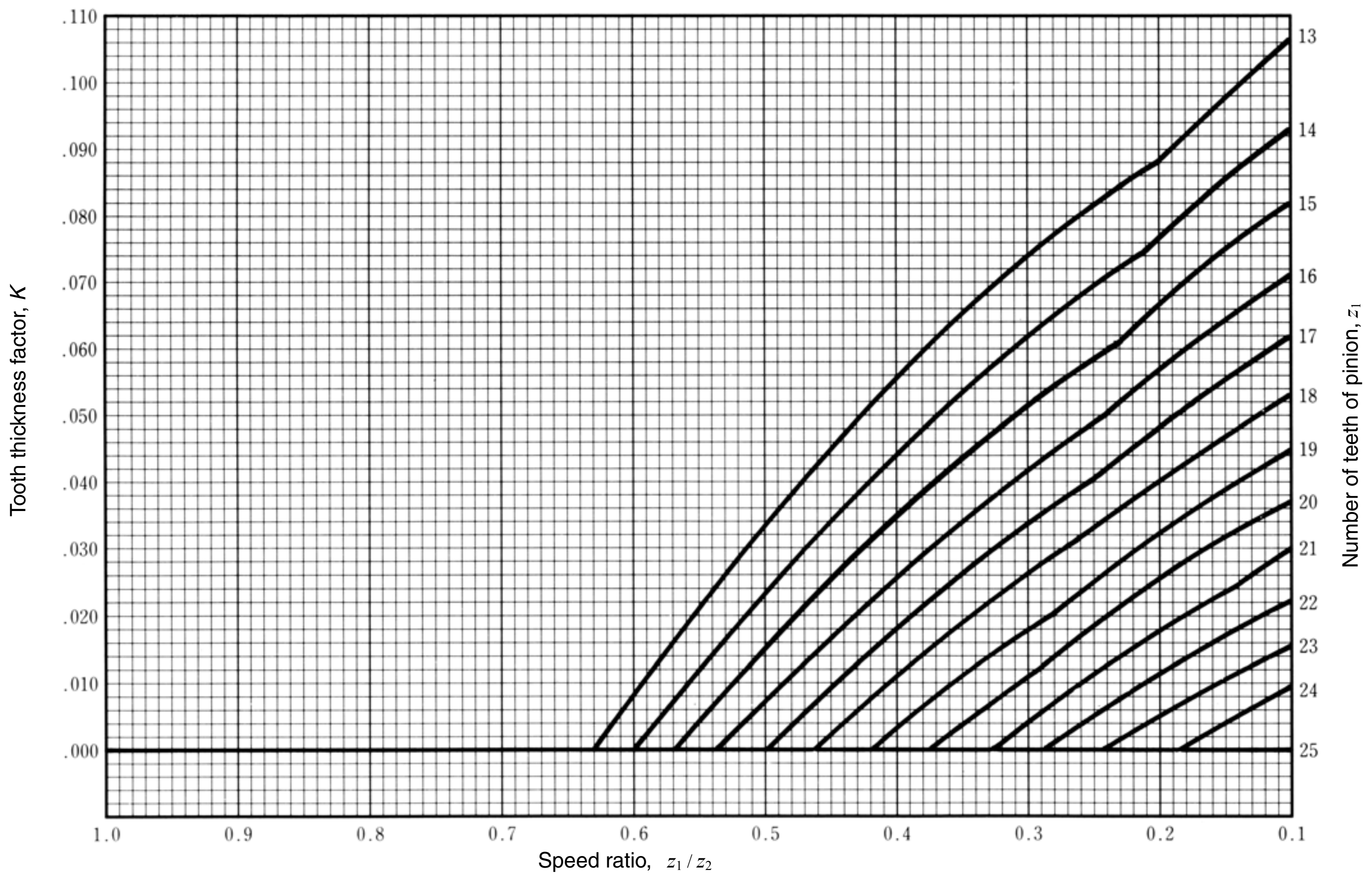


Fig.3.2 Chart to determine the tooth thickness factor k for gleason straight bevel gear

Table 3.6 Equations for chordal tooth thickness of standard straight bevel gears

No.	Item	Symbol	Formula	Example
1	Tooth thickness	s	$\dfrac{\pi m}{2}$	$m = 4$ $\alpha = 20°$ $\Sigma = 90°$ $z_1 = 16$ $z_2 = 40$ $d_1 = 64$ $d_2 = 160$ $h_a = 4.0000$ $\delta_1 = 21.8014°$ $\delta_2 = 68.1986°$ $s = 6.2832$ $z_{v1} = 17.2325$ $z_{v2} = 107.7033$ $R_{v1} = 34.4650$ $R_{v2} = 215.4066$ $\psi_{v1} = 5.2227°$ $\psi_{v2} = 0.83563°$ $\bar{s}_1 = 6.2745$ $\bar{s}_2 = 6.2830$ $\bar{h}_{a1} = 4.1431$ $\bar{h}_{a2} = 4.0229$
2	Number of teeth of an equivalent spur gear	z_v	$\dfrac{z}{\cos\delta}$	
3	Back cone distance	R_v	$\dfrac{d}{2\cos\delta}$	
4	Tooth thickness half angle	ψ_v	$\dfrac{90}{z_v}$	
5	Chordal tooth thickness	$\bar{s}$	$z_v m \sin\psi_v$	
6	Chordal height	$\bar{h}_a$	$h_a + R(1 - \cos\psi_v)$	

If a straight bevel gear is cut by a Gleason straight bevel cutter, the tooth angle should be adjusted according to:

$$\text{Tooth angle } (°) = \frac{180}{\pi R}\left(\frac{s}{2} + h_f \tan\alpha\right) \qquad (3.1)$$

This angle is used as a reference in determining the tooth thickness, s, in setting up the gear cutting machine.

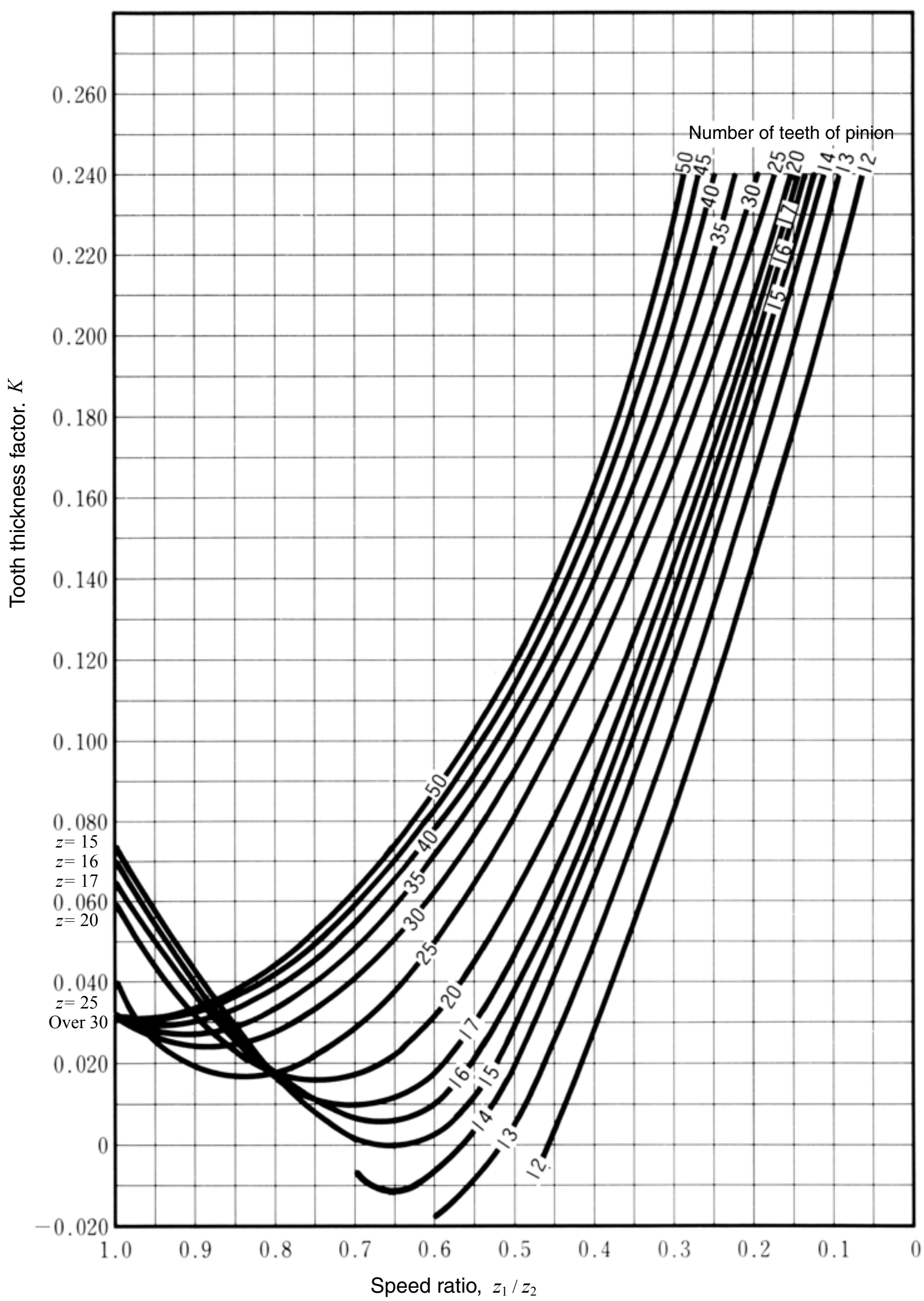


Fig.3.3 Chart to determine the tooth thickness factor k for gleason spiral bevel gears

Table 3.7 Equations for chordal tooth thickness of gleason spiral bevel gears

No.	Item	Symbol	Formula	Example
1	Tooth thickness factor	K	Obtain from Figure 3.3	$\Sigma = 90°$ $m = 3$ $\alpha_n = 20°$ $z_1 = 20$ $z_2 = 40$ $\beta_m = 35°$ $h_{a1} = 3.4275$ $h_{a2} = 1.6725$ $K = 0.060$ $p = 9.4248$ $s_1 = 5.6722$ $s_2 = 3.7526$
2	Tooth thickness	s_1	$p - s_2$	
		s_2	$\frac{p}{2} - (h_{a1} - h_{a2}) \frac{\tan\alpha_n}{\cos\beta_m} - Km$	

The calculations of chordal tooth thickness of a Gleason spiral bevel gear are so complicated that we do not intend to go further in this presentation.

(5) Worm Gear Pair

Table 3.8 presents equations for chordal tooth thickness of axial module worm gear pair. Table 3.9 presents the same of normal module worm gear pair.

Table 3.8 Equations for chordal tooth thickness of axial module worm gear pair

No.	Item	Symbol	Formula	Example
1	Axial tooth thickness of worm	s_{t1}	$\dfrac{\pi m_t}{2}$	$m_t = 3$ $\alpha_n = 20°$ $z_1 = 2$ $\quad z_2 = 30$ $d_1 = 38$ $\quad d_2 = 90$ $a = 65$ $x_{t2} = + 0.33333$ $h_{a1} = 3.0000$ $\quad h_{a2} = 4.0000$ $\gamma = 8.97263°$ $\alpha_t = 20.22780°$ $s_{t1} = 4.71239$ $\quad s_{t2} = 5.44934$ $z_{v2} = 31.12885$ $\psi_{v2} = 3.34335°$ $\bar{s}_1 = 4.6547$ $\quad \bar{s}_2 = 5.3796$ $\bar{h}_{a1} = 3.0035$ $\quad \bar{h}_{a2} = 4.0785$
	Transverse tooth thickness of worm wheel	s_{t2}	$\left(\dfrac{\pi}{2} + 2x_{t2}\tan\alpha_t\right)m_t$	
2	No. of teeth in an equivalent spur gear (Worm wheel)	z_{v2}	$\dfrac{z_2}{\cos^3\gamma}$	
3	Tooth thickness half angle (Worm wheel)	ψ_{v2}	$\dfrac{90}{z_{v2}} + \dfrac{360x_{t2}\tan\alpha_t}{\pi z_{v2}}$	
4	Chordal tooth thickness	$\bar{s}_1$ $\bar{s}_2$	$s_{t1}\cos\gamma$ $z_v m_t \cos\gamma \sin\psi_{v2}$	
5	Chordal height	$\bar{h}_{a1}$ $\bar{h}_{a2}$	$h_{a1} + \dfrac{(s_{t1}\sin\gamma\cos\gamma)^2}{4d_1}$ $h_{a2} + \dfrac{z_v m_t \cos\gamma}{2}(1 - \cos\psi_{v2})$	

Table 3.9 Equations for chordal tooth thickness of normal module worm gear pair

No.	Item	Symbol	Formula	Example
1	Axial tooth thickness of worm	s_{n1}	$\dfrac{\pi m_n}{2}$	$m_n = 3$ $\alpha_n = 20°$ $z_1 = 2$ $\quad z_2 = 30$ $d_1 = 38$ $\quad d_2 = 91.1433$ $a = 65$ $x_{n2} = 0.14278$ $h_{a1} = 3.0000$ $\quad h_{a2} = 3.42835$ $\gamma = 9.08472°$ $s_{n1} = 4.71239$ $\quad s_{n2} = 5.02419$ $z_{v2} = 31.15789$ $\psi_{v2} = 3.07964°$ $\bar{s}_1 = 4.7124$ $\quad \bar{s}_2 = 5.0218$ $\bar{h}_{a1} = 3.0036$ $\quad \bar{h}_{a2} = 3.4958$
	Transverse tooth thickness of worm wheel	s_{n2}	$\left(\dfrac{\pi}{2} + 2x_{n2}\tan\alpha_n\right)m_n$	
2	No. of teeth in an equivalent spur gear (Worm wheel)	z_{v2}	$\dfrac{z_2}{\cos^3\gamma}$	
3	Tooth thickness half angle (Worm gear)	ψ_{v2}	$\dfrac{90}{z_{v2}} + \dfrac{360\,x_{n2}\tan\alpha_n}{\pi z_{v2}}$	
4	Chordal tooth thickness	$\bar{s}_1$ $\bar{s}_2$	s_{n1} $z_{v2} m_n \sin\psi_{v2}$	
5	Chordal height	$\bar{h}_{a1}$ $\bar{h}_{a2}$	$h_{a1} + \dfrac{(s_{n1}\sin\gamma)^2}{4d_1}$ $h_{a2} + \dfrac{z_v m_n}{2}(1 - \cos\psi_{v2})$	

3.2 Span Measurement of Teeth

Span measurement of teeth, W, is a measure over a number of teeth, k, made by means of a special tooth thickness micrometer. The value measured is the sum of normal tooth thickness on the base circle, s_{bn}, and normal pitch, p_{bn} $(k-1)$. See Figure 3.4.

(1) Spur and Internal Gears

The applicable equations are presented in Table 3.10.

Table 3.10 Span measurement of spun and internal gear teeth

No.	Item	Symbol	Formula	Example
1	Span number of teeth	k	$k_{th} = z\,K(f) + 0.5$ See NOTE Select the nearest natural number of z_{mth} as z_m	$m = 3$ $\alpha = 20°$ $z = 24$ $x = +0.4$ $k_{th} = 3.78787$ $k = 4$ $W = 32.8266$
2	Span measurement over k teeth	W	$m \cos\alpha\,\{\pi(k-0.5) + z\,\mathrm{inv}\alpha\} + 2xm \sin\alpha$	

NOTE:

$$K(f) = \frac{1}{\pi}\left\{\sec\alpha\sqrt{(1+2f)^2 - \cos^2\alpha} - \mathrm{inv}\alpha - 2f\tan\alpha\right\} \qquad (3.2)$$

where $f = \dfrac{x}{z}$

Figure 3.4 shows the span measurement of a spur gear. This measurement is on the outside of the teeth.

For internal gears the tooth profile is opposite to that of the external spur gear. Therefore, the measurement is between the inside of the tooth profiles.

(2) Helical Gears

Tables 3.11 and 3.12 present equations for span measurement of the normal and the transverse systems, respectively, of helical gears.

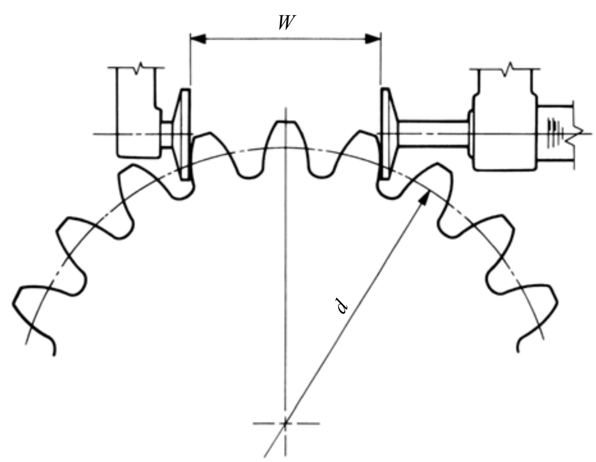


Fig.3.4 Span measurement over k teeth (spur gear)

Table 3.11 Equations for span measurement of the normal system helical gears

No.	Item	Symbol	Formula	Example
1	Span number of teeth	k	$k_{th} = z\,K(f,\beta) + 0.5$ See NOTE Select the nearest natural number of z_{mth} as z_m	$m_n = 3$, $\alpha_n = 20°$, $z = 24$ $\beta = 25°00'00''$ $x_n = +0.4$ $\alpha_t = 21.88023°$ $k_{th} = 4.63009$ $k = 5$ $W = 42.0085$
2	Span measurement over k teeth	W	$m_n \cos\alpha_n\,\{\pi(k-0.5) + z\,\mathrm{inv}\alpha_t\} + 2x_n m_n \sin\alpha_n$	

NOTE:

$$K(f,\beta) = \frac{1}{\pi}\left\{\left(1 + \frac{\sin^2\beta}{\cos^2\beta + \tan^2\alpha_n}\right)\sqrt{(\cos^2\beta + \tan^2\alpha_n)(\sec\beta + 2f)^2 - 1} - \mathrm{inv}\alpha_t - 2f\tan\alpha_n\right\} \qquad (3.3)$$

where $f = \dfrac{x_n}{z}$

Table 3.12 Equations for span measurement of the transverse system helical gears

No.	Item	Symbol	Formula	Example
1	Span number of teeth	k	$k_{th} = z\,K\,(f,\beta) + 0.5$ See NOTE Select the nearest natural number of z_{mth} as z_m	$m_t = 3$, $\alpha_t = 20°$, $z = 24$ $\beta = 22°\ 30'\ 00''$ $x_t = +0.4$ $\alpha_n = 18.58597°$ $k_{th} = 4.31728$ $k = 4$ $W = 30.5910$
2	Span mesurement over k teeth	W	$m_t \cos\beta \cos\alpha_n \{\pi(k - 0.5) + z\,\mathrm{inv}\alpha_t\} + 2x_t m_t \sin\alpha_n$	

NOTE:

$$K(f,\beta) = \frac{1}{\pi}\left\{\left(1 + \frac{\sin^2\beta}{\cos^2\beta + \tan^2\alpha_n}\right)\sqrt{(\cos^2\beta + \tan^2\alpha_n)(\sec\beta + 2f)^2 - 1} - \mathrm{inv}\alpha_t - 2f\tan\alpha_n\right\} \qquad (3.4)$$

where $f = \dfrac{x_t}{z\cos\beta}$

There is a requirement of a minimum facewidth to make a helical gear span measurement. Let b_{min} be the minimum value for facewidth. See Fig.3.5.

Then $b_{min} = W\sin\beta_b + \Delta b$ (3.5)

where β_b is the helix angle at the base cylinder,

$$\left.\begin{aligned}\beta_b &= \tan^{-1}(\tan\beta\cos\alpha_t)\\ &= \sin^{-1}(\sin\beta\cos\alpha_n)\end{aligned}\right\} \qquad (3.6)$$

From the above, we can determine $\Delta b > 3$ mm to make a stable measurement of W.

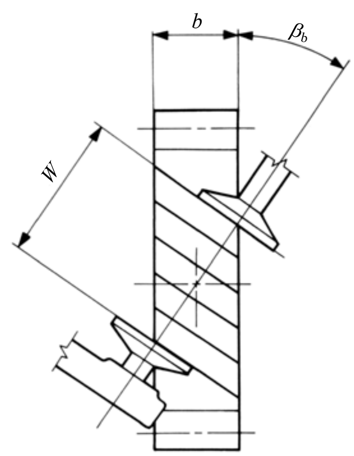
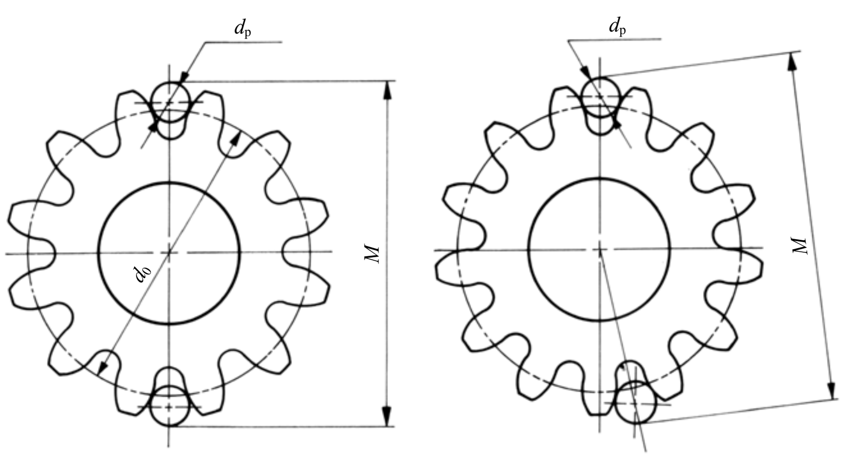


Fig.3.5 Facewidth of helical gear

3.3 Measurement Over Rollers(or generally called over pin/ball measurement)

As shown in Figure 3.6, measurement is made over the outside of two pins that are inserted in diametrically opposite tooth spaces, for even tooth number gears, and as close as possible for odd tooth number gears. The procedure for measuring a rack with a pin or a ball is as shown in Figure 3.8 by putting pin or ball in the tooth space and using a micrometer between it and a reference surface.

Internal gears are similarly measured, except that the measurement is between the pins. See Figure 3.9. Helical gears can only be measured with balls. In the case of a worm, three pins are used, as shown in Figure 3.10. This is similar to the procedure of measuring a screw thread.



Even number of teeth Odd number of teeth

Fig. 3.6 Over pin (ball) measurement

(1) Spur Gears

In measuring a standard gear, the size of the pin must meet the condition that its surface should have the tangent point at the standard pitch circle. While, in measuring a shifted gear, the surface of the pin should have the tangent point at the $d + 2xm$ circle.

Table 3.13 Equations for calculating ideal pin diameters

No.	Item	Symbol	Formula	Example
1	Spacewidth half angle	η	$\left(\frac{\pi}{2z} - \mathrm{inv}\alpha\right) - \frac{2x\tan\alpha}{z}$	$m = 1$ $\alpha = 20°$ $z = 20$ $x = 0$ $\eta = 0.0636354$ $\alpha' = 20°$ $\phi = 0.4276057$ $d'_p = 1.7245$
2	Pressure angle at the point pin is tangent to tooth surface	α'	$\cos^{-1}\left\{\frac{zm\cos\alpha}{(z+2x)m}\right\}$	
3	Pressure angle at pin center	ϕ	$\tan\alpha' + \eta$	
4	Ideal pin diameter	d'_p	$zm\cos\alpha\,(\mathrm{inv}\phi + \eta)$	

NOTE: The units of angles η and ϕ are radians.

The ideal diameters of pins when calculated from the equations of Table 3.13 may not be practical. So, in practice, we select a standard pin diameter close to the ideal value. After the actual diameter of pin d_p is determined, the over pin measurement M can be calculated from Table 3.14.

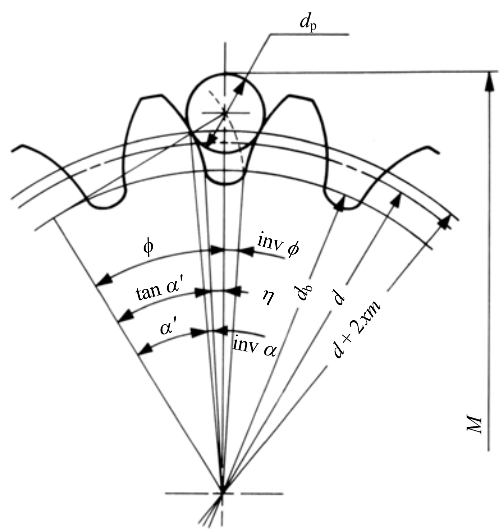


Fig.3.7 Over pins measurement of spur gear

Table 3.14 Equations for over pins measurement for spur gears

No.	Item	Symbol	Formula	Example
1	Pin diameter	d_p	NOTE 1	$d_p = 1.7$ $\mathrm{inv}\phi = 0.0268197$ $\phi = 24.1350°$ $M = 22.2941$
2	Involute function ϕ	inv ϕ	$\frac{d_p}{zm\cos\alpha} - \frac{\pi}{2z} + \mathrm{inv}\alpha + \frac{2x\tan\alpha}{z}$	
3	Pressure angle at pin center	ϕ	Find from involute function table	
4	Measurement over pin (ball)	M	Even teeth $\frac{zm\cos\alpha}{\cos\phi} + d_p$	
			Odd teeth $\frac{zm\cos\alpha}{\cos\phi}\cos\frac{90°}{z} + d_p$	

NOTE: The value of the ideal pin diameter from Table 3.13, or its approximate value, is applied as the actual diameter of pin d_p here.

Table 3.15 is a dimensional table under the condition of module $m = 1$ and pressure angle $\alpha = 20°$ with which the pin has the tangent point at $d + 2xm$ circle.

Table 3.15 The size of pin which has the tangent point at $d + 2xm$ circle of spur gears

$m = 1$, $\alpha = 20°$

Number of teeth z	Profile shift coefficient, x							
	− 0.4	− 0.2	0	0.2	0.4	0.6	0.8	1.0
10		1.6348	1.7886	1.9979	2.2687	2.6079	3.0248	3.5315
20	1.6231	1.6599	1.7245	1.8149	1.9306	2.0718	2.2389	2.4329
30	1.6418	1.6649	1.7057	1.7632	1.8369	1.9267	2.0324	2.1542
40	1.6500	1.6669	1.6967	1.7389	1.7930	1.8589	1.9365	2.0257
50	1.6547	1.6680	1.6915	1.7248	1.7675	1.8196	1.8810	1.9516
60	1.6577	1.6687	1.6881	1.7155	1.7509	1.7940	1.8448	1.9032
70	1.6598	1.6692	1.6857	1.7090	1.7392	1.7759	1.8193	1.8691
80	1.6614	1.6695	1.6839	1.7042	1.7305	1.7625	1.8003	1.8438
90	1.6625	1.6698	1.6825	1.7005	1.7237	1.7521	1.7857	1.8242
100	1.6635	1.6700	1.6814	1.6975	1.7184	1.7439	1.7740	1.8087
110	1.6642	1.6701	1.6805	1.6951	1.7140	1.7372	1.7645	1.7960
120	1.6649	1.6703	1.6797	1.6931	1.7104	1.7316	1.7567	1.7855
130	1.6654	1.6704	1.6791	1.6914	1.7074	1.7269	1.7500	1.7766
140	1.6659	1.6705	1.6785	1.6900	1.7048	1.7229	1.7444	1.7690
150	1.6663	1.6706	1.6781	1.6887	1.7025	1.7195	1.7394	1.7625
160	1.6666	1.6706	1.6777	1.6877	1.7006	1.7164	1.7351	1.7567
170	1.6669	1.6707	1.6773	1.6867	1.6989	1.7138	1.7314	1.7517
180	1.6672	1.6708	1.6770	1.6858	1.6973	1.7114	1.7280	1.7472
190	1.6674	1.6708	1.6767	1.6851	1.6960	1.7093	1.7250	1.7432
200	1.6676	1.6708	1.6764	1.6844	1.6947	1.7074	1.7223	1.7396

(2) Spur Racks and Helical Racks

In measuring a rack, the pin is ideally tangent with the tooth flank at the pitch line. The equations in Table 3.16 can, thus, be derived. In the case of a helical rack, module m, and pressure angle α, in Table 3.16, can be substituted by normal module m_n , and normal pressure angle α_n , resulting in Table 3.16A.

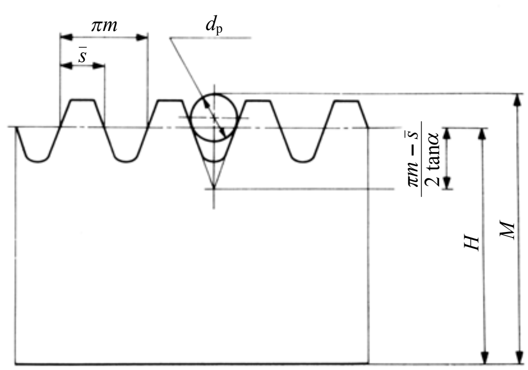


Fig.3.8 Over pins measurement for a rack using a pin or a ball

Table 3.16 Equations for over pins measurement of spur racks

No.	Item	Symbol	Formula	Example
1	Ideal pin diameter	d'_p	$\dfrac{\pi m - \bar{s}}{\cos\alpha}$	$m = 1$ $\alpha = 20°$ $\bar{s} = 1.5708$ $d'_p = 1.6716$ $d_p = 1.7$ $H = 14.0000$ $M = 15.1774$
2	Measurement over pin (ball)	M	$H - \dfrac{\pi m - \bar{s}}{2 \tan\alpha} + \dfrac{d_p}{2}\left(1 + \dfrac{1}{\sin\alpha}\right)$	

Table 3.16A Equations for Over Pins Measurement of Helical Racks

No.	Item	Symbol	Formula	Example
1	Ideal pin diameter	d'_p	$\dfrac{\pi m_n - \bar{s}}{\cos\alpha_n}$	$m_n = 1$ $\alpha_n = 20°$ $\beta = 15°$ $\bar{s} = 1.5708$ $d'_p = 1.6716$ $d_p = 1.7$ $H = 14.0000$ $M = 15.1774$
2	Measurement over pin (ball)	M	$H - \dfrac{\pi m_n - \bar{s}}{2\tan\alpha_n} + \dfrac{d_p}{2}\left(1 + \dfrac{1}{\sin\alpha_n}\right)$	

(3) Internal Gears

As shown in Figure 3.9, measuring an internal gear needs a proper pin which has its tangent point at $d + 2xm$ circle. The equations are in Table 3.17 for obtaining the ideal pin diameter. The equations for calculating the between pin measurement, M, are given in Table 3.18.

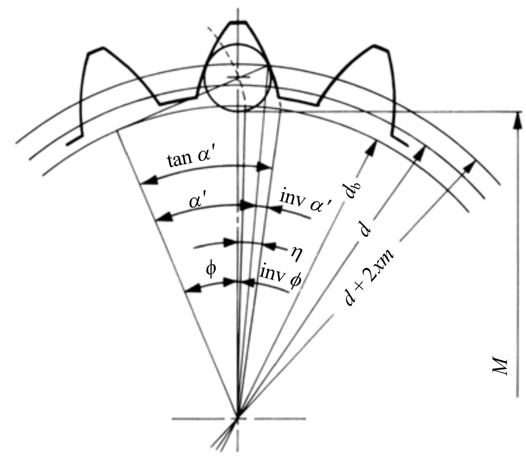


Fig. 3.9 Between pin dimension of internal gears

Table 3.17 Equations for calculating pin diameter for internal gears

No.	Item	Symbol	Formula	Example
1	Spacewidth half angle	η	$\left(\dfrac{\pi}{2z} + \mathrm{inv}\alpha\right) + \dfrac{2x\tan\alpha}{z}$	$m = 1$ $\alpha = 20°$ $z = 40$ $x = 0$ $\eta = 0.054174$ $\alpha' = 20°$ $\phi = 0.309796$ $d'_p = 1.6489$
2	Pressure angle at the point pin is tangent to tooth surface	α'	$\cos^{-1}\left\{\dfrac{zm\cos\alpha}{(z+2x)m}\right\}$	
3	Pressure angle at pin center	ϕ	$\tan\alpha' - \eta$	
4	Ideal pin diameter	d'_p	$zm\cos\alpha\,(\eta - \mathrm{inv}\phi)$	

NOTE: The units of angles η , ϕ are radians.

Tabl 3.18 Equations for between pins measurement of internal gears

No.	Item	Symbol	Formula	Example
1	Pin (ball) diameter	d_p	See NOTE	$d_p = 1.7$ $\mathrm{inv}\phi = 0.0089467$ $\phi = 16.9521°$ $M = 37.5951$
2	Involute function ϕ	inv ϕ	$\left(\dfrac{\pi}{2z} + \mathrm{inv}\alpha\right) - \dfrac{d_p}{zm\cos\alpha} + \dfrac{2x\tan\alpha}{z}$	
3	Pressure angle at pin center	ϕ	Find from involute function table	
4	Between pins measurement	M	Even teeth $\dfrac{zm\cos\alpha}{\cos\phi} - d_p$	
			Odd teeth $\dfrac{zm\cos\alpha}{\cos\phi}\cos\dfrac{90°}{z} - d_p$	

NOTE: First, calculate the ideal pin diameter. Then, choose the nearest practical actual pin size.

Table 3.19 lists ideal pin diameters for standard and profile shifted gears under the condition of module m = 1 and pressure angle α = 20°, which makes the pin tangent to the reference circle $d + 2xm$.

Table 3.19 The size of pin that is tangent at reference circle $d + 2xm$ of internal gears $m = 1$, $\alpha = 20°$

Number of teeth z	Profile shift coefficient, x							
	− 0.4	− 0.2	0	0.2	0.4	0.6	0.8	1.0
10	–	1.4789	1.5936	1.6758	1.7283	1.7519	1.7460	1.7092
20	1.4687	1.5604	1.6284	1.6759	1.7047	1.7154	1.7084	1.6837
30	1.5309	1.5942	1.6418	1.6751	1.6949	1.7016	1.6956	1.6771
40	1.5640	1.6123	1.6489	1.6745	1.6895	1.6944	1.6893	1.6744
50	1.5845	1.6236	1.6533	1.6740	1.6862	1.6900	1.6856	1.6732
60	1.5985	1.6312	1.6562	1.6737	1.6839	1.6870	1.6832	1.6725
70	1.6086	1.6368	1.6583	1.6734	1.6822	1.6849	1.6815	1.6721
80	1.6162	1.6410	1.6600	1.6732	1.6810	1.6833	1.6802	1.6718
90	1.6222	1.6443	1.6612	1.6731	1.6800	1.6820	1.6792	1.6717
100	1.6270	1.6470	1.6622	1.6729	1.6792	1.6810	1.6784	1.6716
110	1.6310	1.6492	1.6631	1.6728	1.6785	1.6801	1.6778	1.6715
120	1.6343	1.6510	1.6638	1.6727	1.6779	1.6794	1.6772	1.6714
130	1.6371	1.6525	1.6644	1.6727	1.6775	1.6788	1.6768	1.6714
140	1.6396	1.6539	1.6649	1.6726	1.6771	1.6783	1.6764	1.6714
150	1.6417	1.6550	1.6653	1.6725	1.6767	1.6779	1.6761	1.6713
160	1.6435	1.6561	1.6657	1.6725	1.6764	1.6775	1.6758	1.6713
170	1.6451	1.6570	1.6661	1.6724	1.6761	1.6772	1.6755	1.6713
180	1.6466	1.6578	1.6664	1.6724	1.6759	1.6768	1.6753	1.6713
190	1.6479	1.6585	1.6666	1.6724	1.6757	1.6766	1.6751	1.6713
200	1.6491	1.6591	1.6669	1.6723	1.6755	1.6763	1.6749	1.6713

(4) Helical Gears

The ideal pin that makes contact at the $d + 2x_n m_n$ reference circle of a helical gear can be obtained from the same above equations, but with the teeth number z substituted by the equivalent (virtual) teeth number z_v.

Table 3.20 presents equations for deriving over pin diameters. Table 3.21 presents equations for calculating over pin measurements for helical gears in the normal system.

Table 3.20 Equations for calculating pin diameter for helical gears in the normal system

No.	Item	Symbol	Formula	Example
1	Number of teeth of an equivalent spur gear	z_v	$\dfrac{z}{\cos^3\beta}$	m_n = 1 α_n = 20° z = 20 β = 15° 00' 00" x_n = + 0.4 z_v = 22.19211 η_v = 0.0427566 α'_v = 24.90647° ϕ_v = 0.507078 d'_p = 1.9020
2	Spacewidth half angle	η_v	$\dfrac{\pi}{2z_v} - \text{inv}\,\alpha_n - \dfrac{2x_n \tan\alpha_n}{z_v}$	
3	Pressure angle at the point pin is tangent to tooth surface	α'_v	$\cos^{-1}\left(\dfrac{z_v \cos\alpha_n}{z_v + 2x_n}\right)$	
4	Pressure angle at pin center	ϕ_v	$\tan\alpha'_v + \eta_v$	
5	Ideal pin diameter	d'_p	$z_v m_n \cos\alpha_n (\text{inv}\phi_v + \eta_v)$	

NOTE: The units of angles η_v and ϕ_v are radians.

Table 3.21 Equations for calculating over pins measurement for helical gears in the normal system

No.	Item	Symbol	Formula	Example
1	Pin (ball) diameter	d_p	See NOTE	Let $d_p = 2$, then α_t = 20.646896° inv ϕ = 0.058890 ϕ = 30.8534° M = 24.5696
2	Involute function φ	inv ϕ	$\dfrac{d_p}{m_n z \cos\alpha_n} - \dfrac{\pi}{2z} + \text{inv}\,\alpha_t + \dfrac{2x_n \tan\alpha_n}{z}$	
3	Pressure angle at pin center	ϕ	Find from involute function table	
4	Measurement over pin (ball)	M	Even Teeth $\dfrac{z m_n \cos\alpha_t}{\cos\beta \cos\phi} + d_p$	
			Odd Teeth $\dfrac{z m_n \cos\alpha_t}{\cos\beta \cos\phi} \cos\dfrac{90°}{z} + d_p$	

NOTE: The ideal pin diameter of Table 3.20, or its approximate value, is entered as the actual diameter of d_p.

Table 3.22 and Table 3.23 present equations for calculating pin measurements for helical gers in the transverse (perpendicular to axis) system.

Table 3.22 Equations for calculating pin diameter for helical gears in the transverse system

No.	Item	Symbol	Formula	Example
1	Number of teeth of an equivalent spur gear	z_v	$\dfrac{z}{\cos^3\beta}$	m_t = 3 α_t = 20° z = 36 β = 33° 33' 26.3" α_n = 16.87300° x_t = + 0.2 z_v = 62.20800 η_v = 0.014091 α'_v = 18.26390 ϕ_v = 0.34411 inv ϕ_v = 0.014258 d'_p = 4.2190
2	Spacewidth half angle	η_v	$\dfrac{\pi}{2z_v} - \text{inv}\,\alpha_n - \dfrac{2x_t \tan\alpha_t}{z_v}$	
3	Pressure angle at the point pin is tangent to tooth surface	α'_v	$\cos^{-1}\left(\dfrac{z_v \cos\alpha_n}{z_v + 2\dfrac{x_t}{\cos\beta}}\right)$	
4	Pressure angle at pin center	ϕ_v	$\tan\alpha'_v + \eta_v$	
5	Ideal pin diameter	d'_p	$z_v m_t \cos\beta \cos\alpha_n (\text{inv}\,\phi_v + \eta_v)$	

NOTE: The units of angles η_v and ϕ_v are radians.

Table 3.23 Equations for calculating over pins measurement for helical gears in the transverse system

No.	Item	Symbol	Formula	Example
1	Pin (ball) diameter	d_p	See NOTE	d_p = 4.5 inv ϕ = 0.027564 ϕ = 24.3453° M = 115.892
2	Involute function ϕ	inv ϕ	$\dfrac{d_p}{m_t z \cos\beta \cos\alpha_n} - \dfrac{\pi}{2z} + \text{inv}\alpha_t + \dfrac{2x_t \tan\alpha_t}{z}$	
3	Pressure angle at pin center	ϕ	Find from involute function table	
4	Measurement over pin (ball)	M	Even teeth $\dfrac{z m_t \cos\alpha_t}{\cos\phi} + d_p$	
			Oddteeth $\dfrac{z m_t \cos\alpha_t}{\cos\phi} \cos\dfrac{90°}{z} + d_p$	

NOTE: The ideal pin diameter of Table 3.22, or its approximate value is applied as the actual diameter of pin d_p here.

(5) Three Wire Method of Worm Measurement

The teeth profile of type III worms which are most popular are cut by standard cutters with a pressure angle $\alpha_n = 20°$. This results in the normal pressure angle of the worm being a bit smaller than 20°. The equation below shows how to calculate a type III worm in an AGMA system.

$$\alpha_n = \alpha_0 - \frac{90}{z_1} \frac{r}{r_0 \cos^2\gamma + r} \sin^3\gamma \qquad (3.7)$$

where r : Worm reference radius
r_0 : Cutter radius
z_1 : Number of threads
γ : Lead angle of worm

The exact equation for a three wire method of type III worm is not only difficult to comprehend, but also hard to calculate precisely. We will introduce two approximate calculation methods here:

(a) Regard the tooth profile of the worm as a straight tooth profile of a rack and apply its equations.

Using this system, the three wire method of a worm can be calculated by Table 3.24.

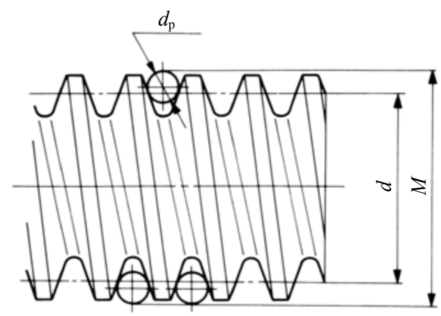


Fig.3.10 Three wire method of a worm

Table 3.24 Equations for three wire method of worm measurement, (a)-1

No.	Item	Symbol	Formula	Example
1	Ideal pin diameter	d'_p	$\dfrac{\pi m_x}{2\cos\alpha_x}$	$m_x = 2$ $\quad \alpha_n = 20°$ $z_1 = 1$ $\quad d_1 = 31$ $\gamma = 3.691386°$ $\alpha_x = 20.03827°$ $d'_p = 3.3440$ d_p Let d_p be 3.3 $M = 35.3173$
2	Three wire measuremnt	M	$d_1 - \dfrac{\pi m_x}{2\tan\alpha_x} + d_p\left(1 + \dfrac{1}{\sin\alpha_x}\right)$	

These equations presume the worm lead angle to be very small and can be neglected. Of course, as the lead angle gets larger, the equations' error gets correspondingly larger. If the lead angle is considered as a factor, the equations are as in Table 3.25.

Table 3.25 Equations for three wire method of worm measurement, (a)-2

No.	Item	Symbol	Formula	Example
1	Ideal pin diameter	d'_p	$\dfrac{\pi m_n}{2\cos\alpha_n}$	$m_x = 2$ $\quad \alpha_n = 20°$ $z_1 = 1$ $\quad d_1 = 31$ $\gamma = 3.691386°$ $m_n = 1.99585$ $d'_p = 3.3363$ d_p Let d_p be 3.3 $M = 35.3344$
2	Three wire measurement	M	$d_1 - \dfrac{\pi m_n}{2\tan\alpha_n} + d_p\left(1 + \dfrac{1}{\sin\alpha_n}\right) - \dfrac{(d_p \cos\alpha_n \sin\gamma)^2}{2 d_1}$	

(b) Consider a worm to be a helical gear.

This means applying the equations for calculating over pins measurement of helical gears to the case of three wire method of a worm. Because the tooth profile of Type III worm is not an involute curve, the method yields an approximation. However, the accuracy is adequate in practice.

Tables 3.26 and 3.27 contain equations based on the axial system. Tables 3.28 and 3.29 are based on the normal system.

Table 3.26 Equation for calculating pin diameter for worms in the axial system

No.	Item	Symbol	Formula	Example
1	Number of teeth of an equivalent spur gear	z_v	$\dfrac{z_1}{\cos^3(90° - \gamma)}$	$m_x = 2$ $\alpha_n = 20°$ $z_1 = 1$ $d_1 = 31$ $\gamma = 3.691386°$ $z_v = 3747.1491$ $\eta_v = -0.014485$ $\alpha'_v = 20°$ $\phi_v = 0.349485$ $\mathrm{inv}\phi_v = 0.014960$ $d'_p = 3.3382$
2	Spacewidth half angle	η_v	$\dfrac{\pi}{2z_v} - \mathrm{inv}\,\alpha_n$	
3	Pressure angle at the point pin is tangent to tooth surface	α'_v	$\cos^{-1}\left(\dfrac{z_v \cos\alpha_n}{z_v}\right)$	
4	Pressure angle at pin center	ϕ_v	$\tan\alpha'_v + \eta_v$	
5	Ideal pin diameter	d'_p	$z_v m_x \cos\gamma \cos\alpha_n (\mathrm{inv}\,\phi_v + \eta_v)$	

NOTE: The units of angles η_v and ϕ_v are radians.

Table 3.27 Equation for three wire method for worms in the axial system

No.	Item	Symbol	Formula	Example
1	Pin (ball) diameter	d_p	See NOTE 1	$d_p = 3.3$ $\alpha_t = 76.96878°$ $\mathrm{inv}\alpha = 4.257549$ $\mathrm{inv}\phi = 4.446297$ $\phi = 80.2959°$ $M = 35.3345$
2	Involute function ϕ	$\mathrm{inv}\,\phi$	$\dfrac{d_p}{m_x z_1 \cos\gamma \cos\alpha_n} - \dfrac{\pi}{2z_1} + \mathrm{inv}\,\alpha_t$	
3	Prssure angle at pin center	ϕ	Find from involute function table	
4	Three wire measurement	M	$\dfrac{z_1 m_x \cos\alpha_t}{\tan\gamma \cos\phi} + d_p$	

NOTE 1. The value of ideal pin diameter from Table 3.26, or its approximate value, is to be used as the actual pin diameter, d_p.

NOTE 2. $\alpha_t = \tan^{-1}\left(\dfrac{\tan\alpha_n}{\sin\gamma}\right)$

Tables 3.28 and 3.29 show the calculation of a worm in the normal module system. Basically, the normal module system and the axial module system have the same form of equations. Only the notations of module make them different.

Table 3.28 Equation for calculating pin diameter for worms in the normal system

No.	Item	Symbol	Formula	Example
1	Number of teeth of an equivalent spur gear	z_v	$\dfrac{z_1}{\cos^3(90° - \gamma)}$	m_n = 2.5 α_n = 20° z_1 = 1 d_1 = 37 γ = 3.874288° z_v = 3241.792 η_v = −0.014420 α'_v = 20° ϕ_v = 0.349550 inv ϕ_v = 0.0149687 d'_p = 4.1785
2	Spacewidth half angle	η_v	$\dfrac{\pi}{2z_v} - \mathrm{inv}\,\alpha_n$	
3	Pressure angle at the point pin is tangent to tooth surface	α'_v	$\cos^{-1}\left(\dfrac{z_v \cos\alpha_n}{z_v}\right)$	
4	Pressure angle at pin center	ϕ_v	$\tan\alpha'_v + \eta_v$	
5	Ideal pin diameter	d'_p	$z_v m_n \cos\alpha_n (\mathrm{inv}\,\phi_v + \eta_v)$	

NOTE: The units of angles η_v and ϕ_v are radians.

Table 3.29 Equations for three wire method for worms in the normal system

No.	Item	Symbol	Formula	Example
1	Pin (ball) diameter	d_p	See NOTE 1.	d_p = 4.2 α_t = 79.48331° invα_t = 3.999514 inv ϕ = 4.216536 ϕ = 79.8947° M = 42.6897
2	Involute function ϕ	inv ϕ	$\dfrac{d_p}{m_n z_1 \cos\alpha_n} - \dfrac{\pi}{2z_1} + \mathrm{inv}\,\alpha_t$	
3	Pressure angle at pin center	ϕ	Find from involute function table	
4	Three wire measurement	M	$\dfrac{z_1 m_n \cos\alpha_t}{\sin\gamma \cos\phi} + d_p$	

NOTE 1. The value of ideal pin diameter from Table 3.28, or its approximate value, is to be used as the actual pin diameter, d_p.

NOTE 2. $\alpha_t = \tan^{-1}\left(\dfrac{\tan\alpha_n}{\sin\gamma}\right)$

4 BACKLASH

Backlash is the amount by which a tooth space exceeds the thickness of a gear tooth engaged in mesh. The general purpose of backlash is to prevent gears from jamming by making contact on both sides of their teeth simultaneously.
there are several kinds of backlash: circumferential backlash j_t, normal backlash j_n, radial backlash j_r and angular backlash j_θ (°), see Figure 4.1.

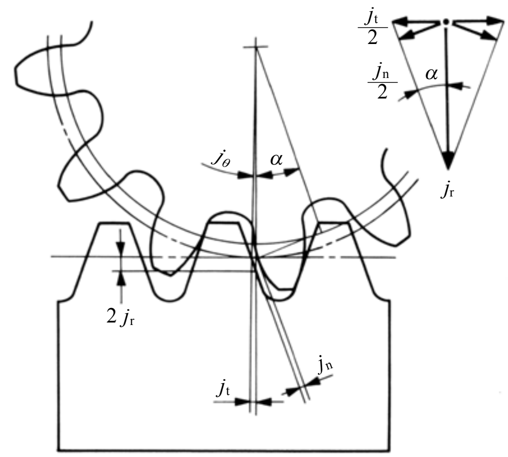


Fig.4.1 Kinds of backlash and their direction

4.1 Backlash Relationships

Table 4.1 reveals relationships among circumferential backlash j_t, normal backlash j_n and radiall backlash j_r .

Table 4.1 The relationships among the backlashes

No.	Type of Gear Meshes	The relation between circumferential backlash j_t and normal backlash j_n	The relation between circumferential backlash j_t and radial backlash j_r
1	Spur gear	$j_n = j_t \cos\alpha$	$j_r = \dfrac{j_t}{2\tan\alpha}$
2	Helical gear	$j_{nn} = j_{tt} \cos\alpha_n \cos\beta$	$j_r = \dfrac{j_{tt}}{2\tan\alpha_t}$
3	Straight bevel gear	$j_n = j_t \cos\alpha$	$j_r = \dfrac{j_t}{2\tan\alpha \sin\delta}$
4	Spiral bevel gear	$j_{nn} = j_{tt} \cos\alpha_n \cos\beta_m$	$j_r = \dfrac{j_{tt}}{2\tan\alpha_t \sin\delta}$
5	Worm	$j_{nn} = j_{tt1} \cos\alpha_n \sin\gamma$	$j_r = \dfrac{j_{tt2}}{2\tan\alpha_x}$
	Worm wheel	$j_{nn} = j_{tt2} \cos\alpha_n \cos\gamma$	

Circumferential backlash j_t has a relation with angular backlash j_θ, as follows:

$$j_\theta = j_t \times \frac{360}{\pi d} \text{ (degrees)} \tag{4.1}$$

(1) Backlash of a Spur Gear Mesh

From Figure 4.1 we can derive backlash of spur gear mesh as:

$$\left.\begin{aligned} j_n &= j_t \cos\alpha \\ j_r &= \frac{j_t}{2\tan\alpha} \end{aligned}\right\} \tag{4.2}$$

(2) Backlash of Helical Gear Mesh

The helical gear has two kinds of backlash when referring to the tooth space. There is a cross section in the normal direction of the tooth surface(n), and a cross section in the radial direction perpendicular to the axis,(t)

j_{nn}= backlash in the direction normal to the tooth surface
j_{tn}= backlash in the circular direction in the cross section normal to the tooth
j_{nt} = backlash in the direction normal to the tooth surface in the cross section perpendicular to the axis
j_{tt} = backlash in the circular direction perpendicular to the axis

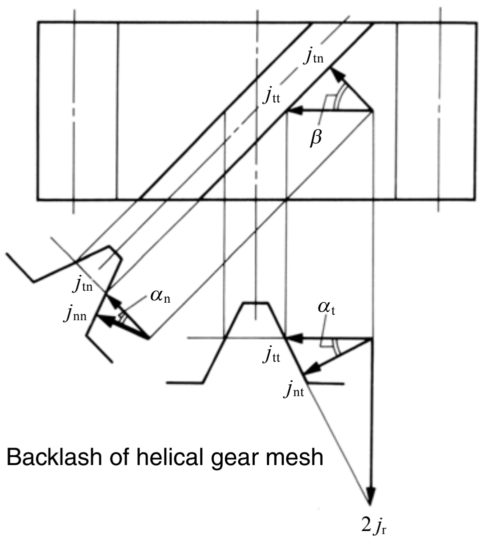


Fig.4.2 Backlash of helical gear mesh

These backlashes have relations as follows:
In the plane normal to the tooth:

$$j_{nn} = j_{tn} \cos\alpha_n \tag{4.3}$$

On the pitch surface:

$$j_{tn} = j_{tt} \cos\beta \tag{4.4}$$

In the plane perpendicular to the axis:

$$\left.\begin{aligned} j_{nt} &= j_{tt} \cos\alpha_t \\ j_r &= \frac{j_{tt}}{2\tan\alpha_t} \end{aligned}\right\} \tag{4.5}$$

(3) Backlash of Straight Bevel Gear Mesh

Figure 4.3 expresses backlash for a straight bevel gear mesh.

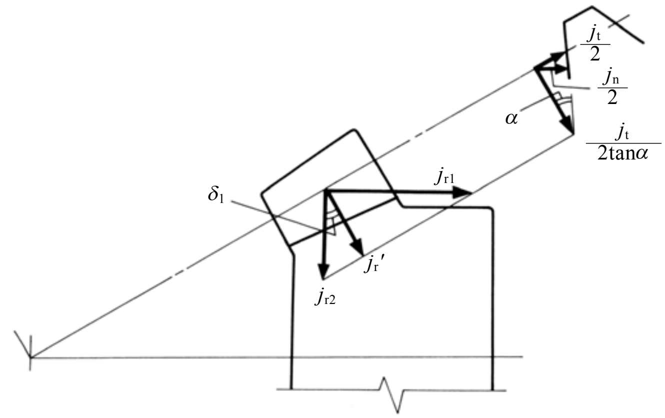


Fig. 4.3 Backlash of straight bevel gear mesh

In the cross section perpendicular to the tooth of a straight bevel gear, circumferential backlash at pitch line j_t , normal backlash j_n and radial backlash j_r' have the following relationships:

$$\left.\begin{aligned} j_n &= j_t \cos\alpha \\ j_r' &= \frac{j_t}{2\tan\alpha} \end{aligned}\right\} \tag{4.6}$$

The radial backlash in the plane of axes can be broken down into the components in the direction of bevel pinion center axis, j_{r1} and in the direction of bevel gear center axis, j_{r2}.

$$\left.\begin{aligned} j_{r1} &= \frac{j_t}{2\tan\alpha \sin\delta_1} \\ j_{r2} &= \frac{j_t}{2\tan\alpha \cos\delta_1} \end{aligned}\right\} \tag{4.7}$$

(4) Backlash of a Spiral Bevel Gear Mesh

Figure 4.4 delineates backlash for a spiral bevel gear mesh.

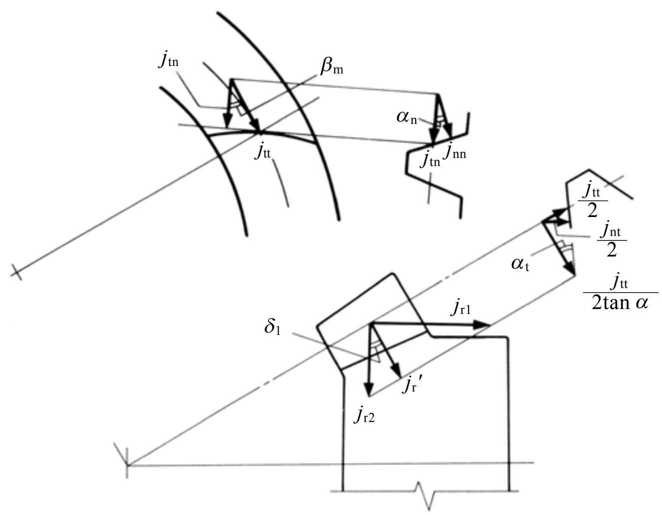


Fig.4.4 Backlash of spiral bevel gear mesh

In the tooth space cross section normal to the tooth:

$$j_{nn} = j_{tn}\cos\alpha_n \qquad (4.8)$$

On the pitch surface

$$j_{tn} = j_{tt}\cos\beta_m \qquad (4.9)$$

In the plane perpendicular to the generatrix of the pitch cone:

$$\left.\begin{aligned} j_{nt} &= j_{tt}\cos\alpha_t \\ j_r' &= \frac{j_{tt}}{2\tan\alpha_t} \end{aligned}\right\} \qquad (4.10)$$

The transverse backlash in the plane of axes j_r' can be broken down into the components in the direction of bevel pinion center axis, j_{r1}, and in the direction of bevel gear center axis, j_{r2}.

$$\left.\begin{aligned} j_{r1} &= \frac{j_{tt}}{2\tan\alpha_t\sin\delta_1} \\ j_{r2} &= \frac{j_{tt}}{2\tan\alpha_t\cos\delta_1} \end{aligned}\right\} \qquad (4.11)$$

(5) Backlash of Worm Gear Pair Mesh

Figure 4.5 expresses backlash for a worm gear pair mesh.

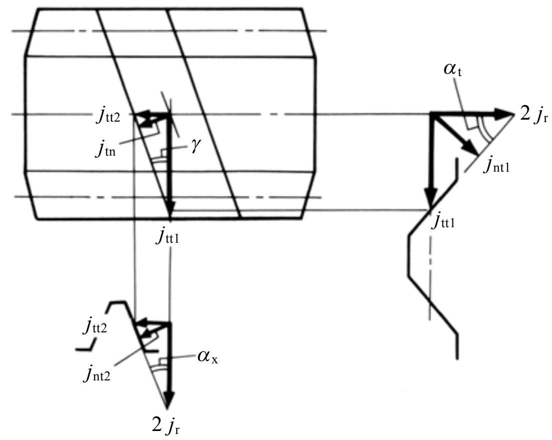


Fig.4.5 Backlash of worm gear pair

On the pitch surface of a worm:

$$\left.\begin{aligned} j_{tn} &= j_{tt1}\sin\gamma \\ j_{tn} &= j_{tt2}\cos\gamma \\ \tan\gamma &= \frac{j_{tt2}}{j_{tt1}} \end{aligned}\right\} \qquad (4.12)$$

In the cross section of a worm perpendicular to its axis:

$$\left.\begin{aligned} j_{nt1} &= j_{tt1}\cos\alpha_t \\ j_r &= \frac{j_{tt1}}{2\tan\alpha_t} \end{aligned}\right\} \qquad (4.13)$$

In the plane perpendicular to the axis of the worm wheel:

$$\left.\begin{aligned} j_{nt2} &= j_{tt2}\cos\alpha_x \\ j_r &= \frac{j_{tt2}}{2\tan\alpha_x} \end{aligned}\right\} \qquad (4.14)$$

4.2 Tooth Thickness and Backlash

There are two ways to produce backlash. One is to enlarge the center distance. The other is to reduce the tooth thickness. The latter is much more popular than the former. We are going to discuss more about the way of reducing the tooth thickness.

In SECTION 3, we have discussed the standard tooth thickness s_1 and s_2. In the meshing of a pair of gears, if the tooth thickness of pinion and gear were reduced by Δs_1 and Δs_2, they would produce a backlash of Δs_1 and Δs_2 in the direction of the pitch circle. Let the magnitude of Δs_1 and Δs_2 be 0.1. We know that $\alpha = 20°$, then:

$$j_t = \Delta s_1 + \Delta s_2$$
$$= 0.1 + 0.1 = 0.2$$

We can convert it into the backlash on normal direction j_n:

$$j_n = j_t \cos \alpha$$
$$= 0.2 \times \cos 20 = 0.1879$$

Let the backlash on the center distance direction be j_r, then:

$$j_r = \frac{j_t}{2 \tan \alpha}$$
$$= \frac{0.2}{2 \times \tan 20°} = 0.2747$$

They express the relationship among several kinds of backlashes. In application, one should consult the JIS standard. There are two JIS standards for backlash – one is JIS B 1703-76 for spur gears and helical gears, and the other is JIS B 1705-73 for bevel gears. All these standards regulate the standard backlashes in the direction of the pitch circle j_t or j_{tt}. These standards can be applied directly, but the backlash beyond the standards may also be used for special purposes. When writing tooth thicknesses on a drawing, it is necessary to specify, in addition, the tolerances on the thicknesses as well as the backlash. For example:

Tooth thickness	$3.141 \ ^{-0.050}_{-0.100}$
Backlash	0.100 ～ 0.200

Since the tooth thickness directly relates to backlash, the tolerances on the thickness will become a very important factor.

4.3 Gear Train and Backlash

The discussions so far involved a single pair of gears. Now, we are going to discuss two stage gear trains and their backlash. In a two stage gear train, as Figure 4.6 shows, j_{t1} and j_{t4} represent the backlashes of first stage gear train and second stage gear train respectively.

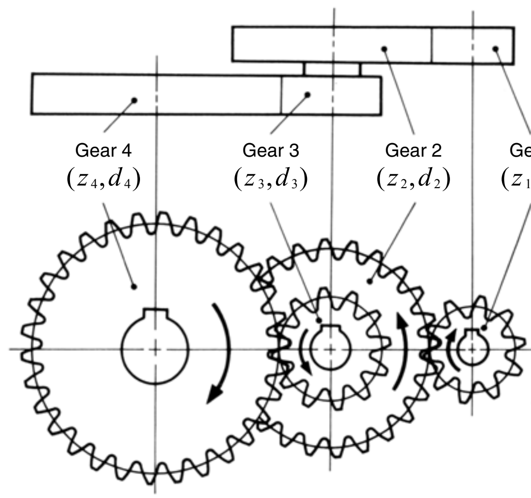


Fig.4.6 Overall accumulated backlash of two stage gear train

If number one gear were fixed, then the accumulated backlash on number four gear j_{tT4} would be as follows:

$$j_{tT4} = j_{t1} \frac{d_3}{d_2} + j_{t4} \tag{4.15}$$

This accumulated backlash can be converted into rotation in degrees:

$$j_\theta = j_{tT4} \frac{360}{\pi d_4} \text{ (degrees)} \tag{4.16}$$

The reverse case is to fix number four gear and to examine the accumulated backlash on number one gear j_{tT1}.

$$j_{tT1} = j_{t4} \frac{d_2}{d_3} + j_{t1} \tag{4.17}$$

This accumulated backlash can be converted into rotation in degrees:

$$j_\theta = j_{tT1} \frac{360}{\pi d_1} \text{ (degrees)} \tag{4.18}$$

4.4 Methods of Controlling Backlash

In order to meet special needs, precision gears are used more frequently than ever before. Reducing backlash becomes an important issue. There are two methods of reducing or eliminating backlash – one a static, and the other a dynamic method. The static method concerns means of assembling gears and then making proper adjustments to achieve the desired low backlash. The dynamic method introduces an external force which continually eliminates all backlash regardless of rotational position.

(1) Static Method

This involves adjustment of either the gear's effective tooth thickness or the mesh center distance. These two independent adjustments can be used to produce four possible combinations as shown in Table 4.2.

Table 4.2 The combination of adjustment

		Center Distance	
		Fixed	Adjustable
Gear Size	Fixed	A	C
	Adjustable	B	D

(A) Case A

By design, center distance and tooth thickness are such that they yield the proper amount of desired minimum backlash. Center distance and tooth thickness size are fixed at correct values and require precision manufacturing.

(B) Case B

With gears mounted on fixed centers, adjustment is made to the effective tooth thickness by axial movement or other means. Three main methods are:

①Two identical gears are mounted so that one can be rotated relative to the other and fixed. In this way,the effective tooth thickness can be adjusted to yield the desired low backlash.

②A gear with a helix angle such as a helical gear is made in two half thicknesses. One is shifted axially such that each makes contact with the mating gear on the opposite sides of the tooth.

③The backlash of cone shaped gears, such as bevel and tapered tooth spur gears, can be adjusted with axial positioning. A duplex lead worm can be adjusted similarly.

Figure 4.7 delineate these three methods.

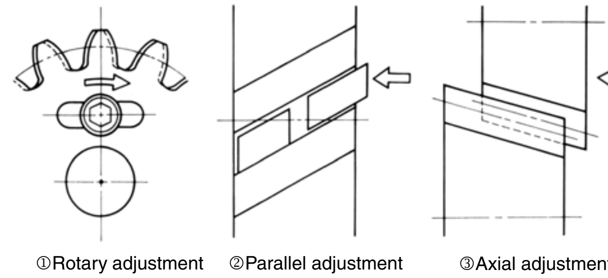


Fig.4.7 Ways of decreasing backlash in case B

(C) Case C

Center distance adjustment of backlash can be accomplished in two ways:

① Linear Movement

Figure 4.8①shows adjustment along the line-of-centers in a straight or parallel axes manner. After setting to the desired value of backlash, the centers are locked in place.

② Rotary Movement

Figure 4.8 ②shows an alternate way of achieving center distance adjustment by rotation of one of the gear centers by means of a swing arm on an eccentric bushing. Again, once the desired backlash setting is found, the positioning arm is locked.

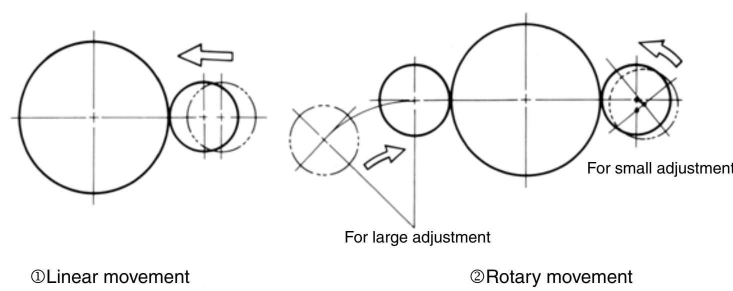


Fig.4.8 Ways of decreasing backlash in case C

(D) Case D

Adjustment of both center distance and tooth thickness is theoretically valid, but is not the usual practice. This would call for needless fabrication expense.

(2) Dynamic Methods

Dynamic methods relate to the static techniques. However, they involve a forced adjustment of either the effective tooth thickness or the center distance.

(A) Backlash Removal by Forced Tooth Contact

This is derived from static Case B. Referring to Figure 4.7①, a forcing spring rotates the two gear halves apart. This results in an effective tooth thickness that continually fills the entire tooth space in all mesh positions.

(B) Backlash Removal by Forced Center Distance Closing

This is derived from static Case C. A spring force is applied to close the center distance; in one case as a linear force along the line-of-centers, and in the other case as a torque applied to the swing arm.

In all of these dynamic methods, the applied external force should be known and properly specified. The theoretical relationship of the forces involved is as follows:

$$F > F_1 + F_2 \tag{4.19}$$

where: F_1 = Transmission Load on Tooth Surface
F_2 = Friction Force on Tooth Surface

If $F < F_1 + F_2$, then it would be impossible to remove backlash. But if F is excessively greater than a proper level, the tooth surfaces would be needlessly loaded and could lead to premature wear and shortened life. Thus, in designing such gears, consideration must be given to not only the needed transmission load, but also the forces acting upon the tooth surfaces caused by the spring load.

(3) Duplex Lead Worm Gear Pair

A duplex lead worm gear mesh is a special design in which backlash can be adjusted by shifting the worm axially. It is useful for worm drives in high precision turntables and hobbing machines. Figure 4.9 presents the basic concept of a duplex lead worm gear pair.

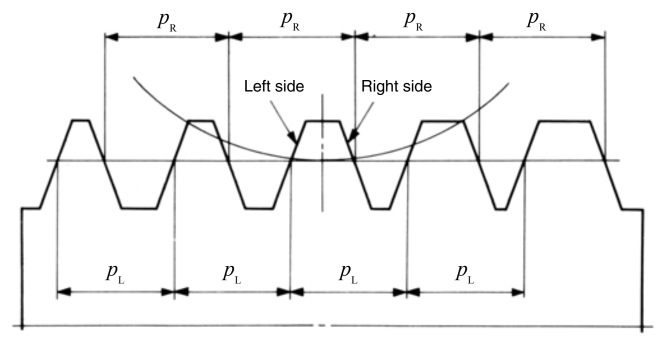


Fig.4.9 Basic concept of duplex lead worm gear pair

The lead or pitch, p_L and p_R , on the two sides of the worm thread are not identical. The example in Figure 4.9 shows the case when $p_R > p_L$. To produce such a worm wheel requires a special dual lead hob. The intent of Figure 4.9 is to indicate that the worm tooth thickness is progressively bigger towards the right end. Thus, it is convenient to adjust backlash by simply moving the duplex worm in the axial direction.

5 GEAR ACCURACY

Gears are one of the basic elements used to transmit power and position. As designers, we desire them to meet various demands:

①Maximum power capability

②Minimum size.

③Minimum noise (silent operation).

④Accurate rotation/position

To meet various levels of these demands requires appropriate degrees of gear accuracy. This involves several gear features.

5.1 Accuracy of Spur and Helical Gears

JIS B 1702-01: 1998 and JIS B 1702-02: 1998 prescribe gear accuracy on spur and helical gears. These two revises the previous specification JIS B 1702: 1976. which described 9 grades grouped from 0 through 8. In order to avoid confusion between old and new specifications, each grades in the revised JIS B 1702 has a prefix 'N', like N4 grade and N10 grade etc.

JIS B 1702-1:1998 Cylindrical gears - gear accuracy - Part 1: Definitions and allowable values of deviations relevant to corresponding flanks of gear teeth. (This specification describes 13 grades of gear accuracy grouped from 0 through 12, - 0, the highest grade and 12, the lowest grade).

JIS B 1702-2:1998 Cylindrical gears - gear accuracy - Part 2: Definitions and allowable values of deviations relevant to radial composite deviations and runout information. (This specification consists of 9 grades of gear accuracy grouped from 4 through 12, - 4, the highest grade and 12, the lowest grade).

Single Pitch Deviation (f_{pt})

The deviation between actual measured pitch value between any adjacent tooth surface and theoretical circular pitch.

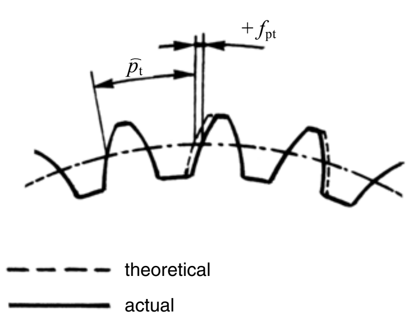


Fig.5.1 Single pitch deviation f_{pt}

Total Cumulative Pitch Deviation (F_p)

Difference between theoretical summation over any number of teeth interval, and summation of actual pitch measurement over the same interval.

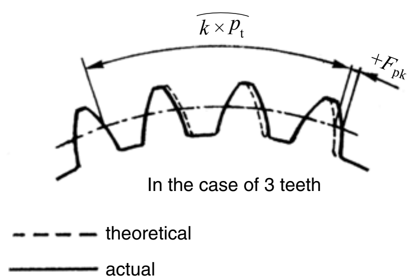


Fig.5.2 Total cumulative pitch deviation (f_p)

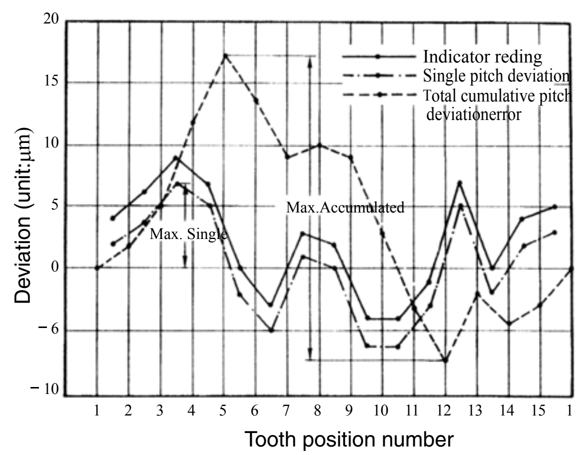


Fig.5.3 Examples of pitch deviation for a 15 tooth gear

Total Profile Deviation (F_α)

Total profile deviation represents the distance (F_α) shown in Figure 5.4. Actual profile chart is lying in between upper design chart and lower design chart.

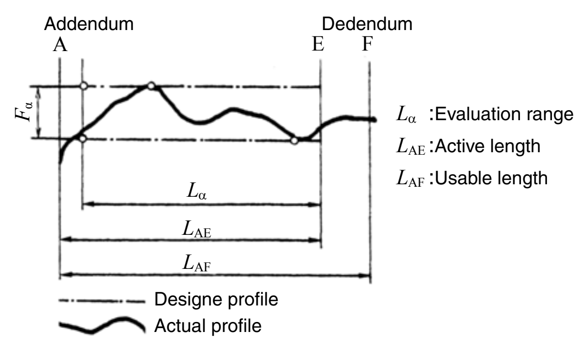


Fig.5.4 Total profile deviation f_α

Total Helix Deviation (F_β)

Total helix deviation represents the distance (F_β) shown in Figure 5.5. The actual helix chart is lying in between upper helix chart and lower helix chart. Total helix deviation results in poor tooth contact, particularly concentrating contact to the tip area. Modifications, such as tooth crowning and end relief can alleviate this deviation to some degree. Shown in Figure 5.6 is an example of a chart measuring total profile deviation and total helix deviation using a Zeiss UMC 550 tester.

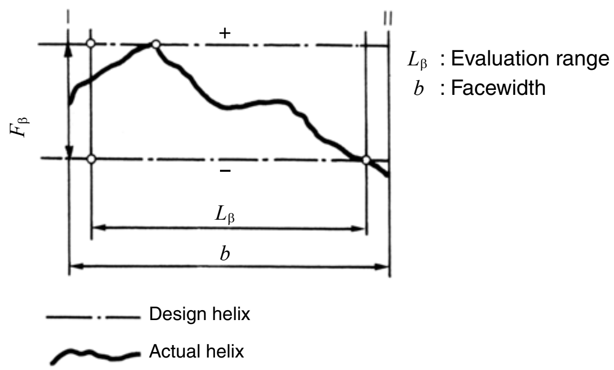


Fig.5.5 Total helix deviation (F_β)

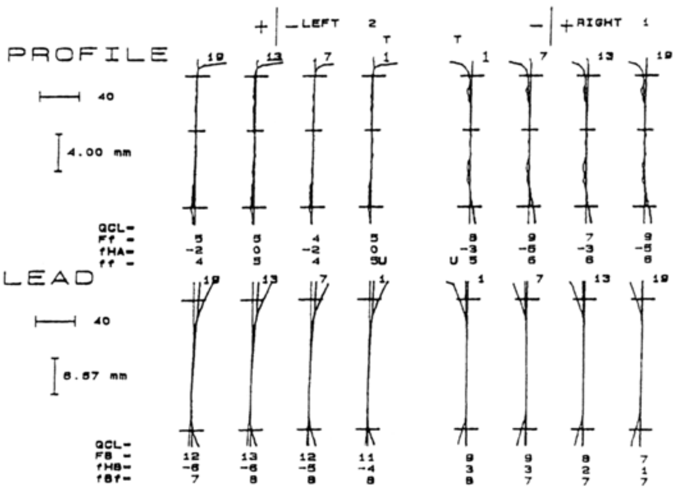


Fig.5.6 An example of a chart measuring total profile deviation and total helix deviation

Total Radial Composite Deviation(F_i'')

Total radial composite deviation represents variation in center distance when product gear is rotated one revolution in tight mesh with a master gear.

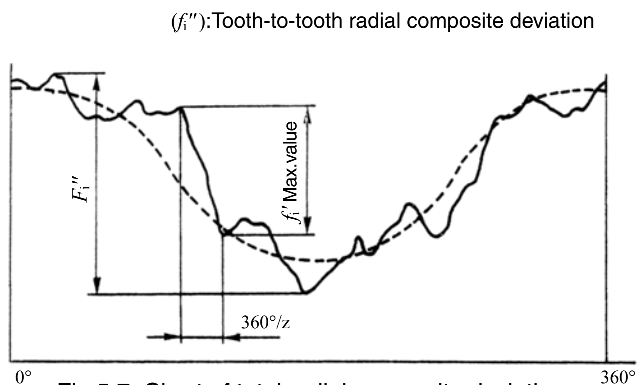


Fig.5.7 Chart of total radial composite deviation

Runout Error of Gear Teeth (F_r)

Most often runout error is mesured by indicating the position of a pin or ball inserted in each tooth space around the gear and taking the largest difference.

Runout causes a number of problems, one of which is noise. The source of this error is most often insufficient accuracy and ruggedness of the cutting arbor and tooling system. And, therefore, it is very importnt to pay attention to these cutting arbor and tooling system to reduce runout error. Shown in Fig.5.8 is the chart of runout. The values of runout includes eccentricity.

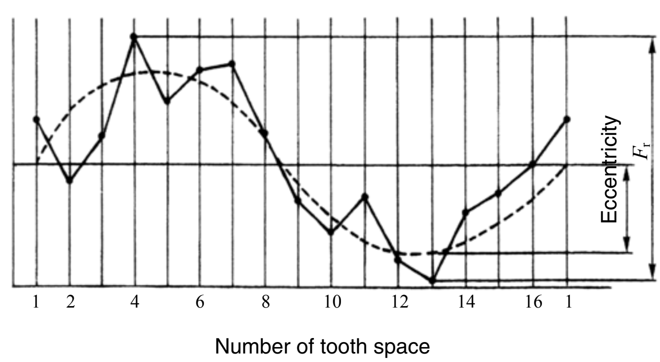


Fig.5.8 Runout error of a 16-tooth gear

5.2 Accuracy of Bevel Gears

JIS B 1704:1978 regulates the specification of a bevel gear's accuracy. It also groups bevel gears into 9 grades, from 0 to 8.

There are 4 types of allowable errors:

①Single pitch error.

②Pitch variation error

③Accumulated pitch error.

④Runout error of teeth (pitch circle).

These are similar to the spur gear errors.

① Single pitch error

The deviation between actual measured pitch value between any adjacent teeth and the theoretical circular pitch at the mean cone distance.

② Pitch variation error

Absolute pitch variation between any two adjacent teeth at the mean cone distance.

③ Accumulated pitch error

Difference between theoretical pitch sum of any teeth interval, and the summation of actual measured pitches for the same teeth interval at the mean cone distance.

④ Runout error of teeth

This is the maximum amount of tooth runout in the radial direction, measured by indicating a pin or ball placed between two teeth at the central cone distance.

Table 5.1 presents equations for allowable values of these various errors.

Table 5.1 Equations for allowable single pitch error, accumulated pitch error and pitch cone runout error, (μm)

Grade	Single pitch error	Accumulated pitch error	Runout error of pitch cone
JIS 0	$0.4W + 2.65$	$1.6W + 10.6$	$2.36\sqrt{d}$
1	$0.63W + 5.0$	$2.5W + 20.0$	$3.6\sqrt{d}$
2	$1.0W + 9.5$	$4.0W + 38.0$	$5.3\sqrt{d}$
3	$1.6W + 18.0$	$6.4W + 72.0$	$8.0\sqrt{d}$
4	$2.5W + 33.5$	$10.0W + 134.0$	$12.0\sqrt{d}$
5	$4.0W + 63.0$	–	$18.0\sqrt{d}$
6	$6.3W + 118.0$	–	$27.0\sqrt{d}$
7	–	–	$60.0\sqrt{d}$
8	–	–	$130.0\sqrt{d}$

where W : Tolerance unit

$$W = \sqrt[3]{d} + 0.65m \ (\mu\text{m})$$

d : Reference Diameter (mm)

The equations of allowable pitch variations are in Table 5.2.

Table 5.2 The Formula of allowable pitch variation error (μm)

Single pitch error, k	Pitch variation error
Less then 70	$1.3k$
70 or more,but less than 100	$1.4k$
100 or more,but less than 150	$1.5k$
More than 150	$1.6k$

Besides the above errors, there are seven specifications for bevel gear blank dimensions and angles, plus an eighth that concerns the cut gear set:

① The tolerance of the blank tip diameter and the crown to back surface distance.

② The tolerance of the outer cone angle of the gear blank.

③ The tolerance of the cone surface runout of the gear blank.

④ The tolerance of the side surface runout of the gear blank.

⑤ The feeler gauze size to check the flatness of blank back surface.

⑥ The tolerance of the shaft runout of the gear blank.

⑦ The tolerance of the shaft bore dimension deviation of the gear blank.

⑧ The tooth contact.

Item 8 relates to cutting of the two mating gears' teeth. The tooth contact must be full and even across the profiles. This is an important criterion that supersedes all other blank requirements.

5.3 Running (Dynamic) Gear Testing

An alternate simple means of testing the general accuracy of a gear is to rotate it with a mate, preferably of known high quality, and measure characteristics during rotation. This kind of tester can be either single contact (fixed center distance method) or dual (variable center distance method). This refers to action on one side or simultaneously on both sides of the tooth. This is also commonly referred to as single and double flank testing. Because of simplicity, dual contact testing is more popular than single contact.

(1) Dual Contact (Double Flank) Testing

In this technique, the gear is forced meshed with a master gear such that there is intimate tooth contact on both sides and, therefore, no backlash. The contact is forced by a loading spring. As the gears rotate, there is variation of center distance due to various errors, most notably runout. This variation is measured and is a criterion of gear quality. A full rotation presents the total gear error, while rotation through one pitch is a tooth-to-tooth error. Figure 5.9 presents a typical plot for such a test.

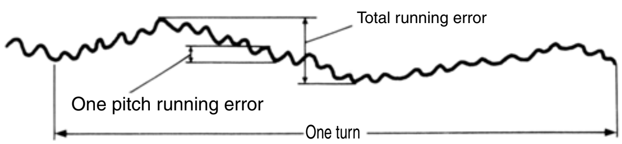


Fig.5.9 Example of dual contact running testing report

Allowable errors per JGMA 116-01 are presented in Table 5.3.

Table 5.3 Allowable values of running errors, (μm)

Grade	Tooth-to-tooth composite error	Total composite error
Extra fine (0)	$1.12m + 3.55$	$(1.4W + 4.0) + 0.5(1.12m + 3.55)$
1	$1.6m + 5.0$	$(2.0W + 5.6) + 0.5(1.6m + 5.0)$
2	$2.24m + 7.1$	$(2.8W + 8.0) + 0.5(2.24m + 7.1)$
3	$3.15m + 10.0$	$(4.0W + 11.2) + 0.5(3.15m + 10.0)$
4	$4.5m + 14.0$	$(5.6W + 16.0) + 0.5(4.5m + 14.0)$
5	$6.3m + 20.0$	$(8.0W + 22.4) + 0.5(6.3m + 20.0)$
6	$9.0m + 28.0$	$(11.2W + 31.5) + 0.5(9.0m + 28.0)$
7	$12.5m + 40.0$	$(22.4W + 63.0) + 0.5(12.5m + 40.0)$
8	$18.0m + 56.0$	$(45.0W + 125.0) + 0.5(18.0m + 56.0)$

where W : Tolerance unit

$W = \sqrt[3]{d} + 0.65m$ (μm)

d : Reference diameter (mm)

m : Module (mm)

(2) Single Contact Testing

In this test, the gear is mated with a master gear on a fixed center distance and set in such a way that only one tooth side makes contact. The gears are rotated through this single flank contact action, and the angular transmission error of the driven gear is measured. This is a tedious testing method and is seldom used except for inspection of the very highest precision gears.

6 FEATURES OF TOOTH CONTACT

Tooth contact is critical to noise, vibration, efficiency, strength, wear and life. To obtain good contact, the designer must give proper consideration to the following features:

- Modifying the tooth shape
  Improve tooth contact by crowning or end relief.

- Using higher precision gear
  Specify higher accuracy by design. Also, specify that the manufacturing process is to include grinding or lapping.

- Controlling the accuracy of the gear assembly
  Specify adequate shaft parallelism and perpendicularity of the gear housing (box or structure)

Tooth contact of spur and helical gears can be reasonably controlled and verified through piece part inspection. However, for the most part, bevel gears and worm gear pair cannot be equally well inspected. Consequently, final inspection of bevel and worm mesh tooth contact in assembly provides a quality criterion for control. Then, as required, gears can be axially adjusted to achieve desired contact.

JIS B 1741: 1977 classifies tooth contact into three levels, as presented in Table 6.1.

Table 6.1 Levels of tooth contact

Level	Types of gear	Levels of tooth contact	
		Tooth width direction	Tooth height direction
A	Cylindrical gears	More than 70%	More than 40%
	Bevel gears	More than 50%	
	Worm wheels		
B	Cylindrical gears	More than 50%	More than 30%
	Bevel gears	More than 35%	
	Worm wheels		
C	Cylindrical gears	More than 35%	More than 20%
	Bevel gears	More than 25%	
	Worm wheels	More than 20%	

The percentage in Table 6.1 considers only the effective width and height of teeth.

6.1 Tooth Contact of a Bevel Gear

It is important to check the tooth contact of a bevel gear both during manufacturing and again in final assembly. The method is to apply a colored dye and observe the contact area after running. Usually some load is applied, either the actual or applied braking, to realize a realistic contact condition. Ideal contact favors the toe end under no or light load, as shown in Figure 6.1; and, as load is increased to full load, contact shifts to the central part of the tooth width.

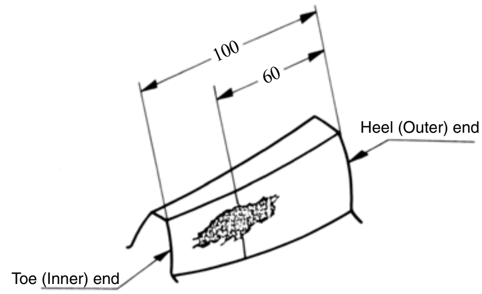


Fig.6.1 Central toe contact

Even when a gear is ideally manufactured, it may reveal poor tooth contact due to lack of precision in housing or improper mounting position, or both. Usual major faults are:

①Shafts are not intersecting, but are skew (offset error)

②Shaft angle error of gearbox.

③Mounting distance error.

Errors① and ② can be corrected only by reprocessing the housing/mounting. Error ③ can be corrected by adjusting the gears in an axial direction. All three errors may be the cause of improper backlash.

(1) The Offset Error of Shaft Alignment

If a gearbox has an offset error, then it will produce crossed contact, as shown in Figure 6.2. This error often appears as if error is in the gear tooth orientation.

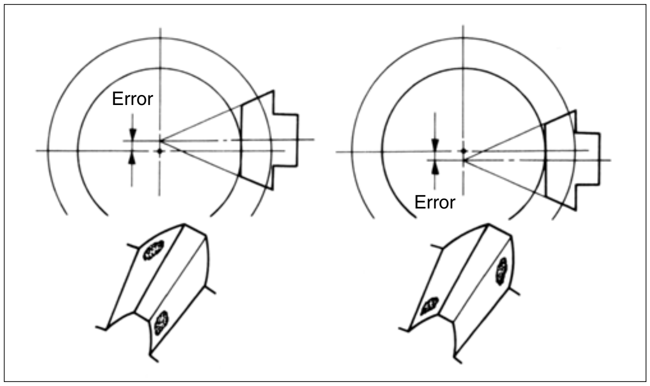


Fig.6.2 Poor tooth contact due to offset error of shafts

(2) The Shaft Angle Error of Gear Box

As Figure 6.3 shows, the tooth contact will move toward the toe end if the shaft angle error is positive; the tooth contact will move toward the heel end if the shaft angle error is negative.

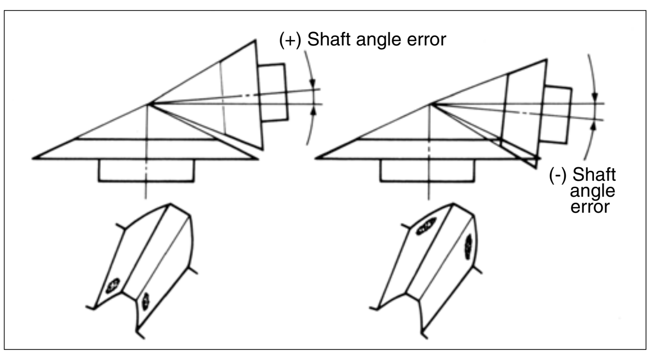


Fig.6.3 Poor tooth contact due to shaft angle error

(3) Mounting Distance Error

When the mounting distance of the pinion is a positive error, the contact of the pinion will move towards the tooth root, while the contact of the mating gear will move toward the top of the tooth. This is the same situation as if the pressure angle of the pinion is smaller than that of the gear. On the other hand, if the mounting distance of the pinion has a negative error, the contact of the pinion will move toward the top and that of the gear will move toward the root. This is similar to the pressure angle of the pinion being larger than that of the gear. These errors may be diminished by axial adjustment with a backing shim.

The various contact patterns due to mounting distance errors are shown in Figure 6.4.

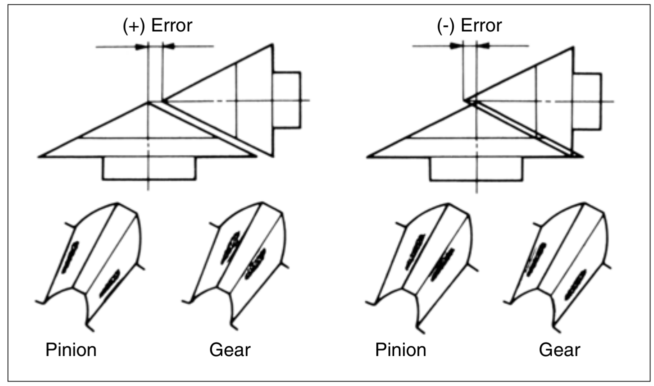


Fig.6.4 Poor tooth contact due to error in mounting distance

Mounting distance error will cause a change of backlash; positive error will increase backlash; and negative, decrease. Since the mounting distance error of the pinion affects the tooth contact greatly, it is customary to adjust the gear rather than the pinion in its axial direction.

6.2 Tooth Contact of a Worm Gear Pair

There is no specific Japanese standard concerning worm gearing, except for some specifications regarding tooth contact in JIS B 1741: 1977.

Therefore, it is the general practice to test the tooth contact and backlash with a tester. Figure 6.5 shows the ideal contact for a worm mesh.

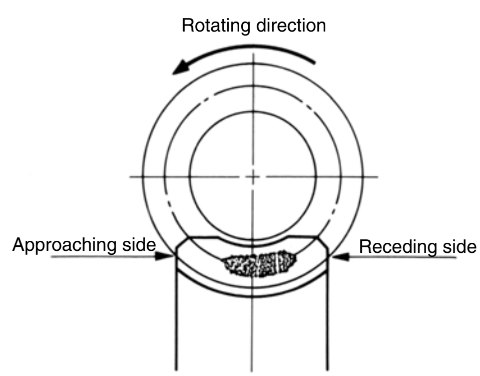


Fig.6.5 Ideal tooth contact of worm gear pair

From Figure 6.5, we realize that the ideal portion of contact inclines to the receding side.

Because the clearance in the approaching side is larger than in the receding side, the oil film is established much easier in the approaching side. However, an excellent worm wheel in conjunction with a defective gearbox will decrease the level of tooth contact and the performance. There are three major factors, besides the gear itself, which may influence the tooth contact:

① Shaft Angle Error.
② Center Distance Error.
③ Locating Distance Error of Worm Wheel.

Errors① and ② can only be corrected by remaking the housing. Error③ may be decreased by adjusting the worm wheel along the axial direction. These three errors introduce varying degrees of backlash.

(1) Shaft Angle Error

If the gear box has a shaft angle error, then it will produce crossed contact as shown in Figure 6.6.
A helix angle error will also produce a similar crossed contact.

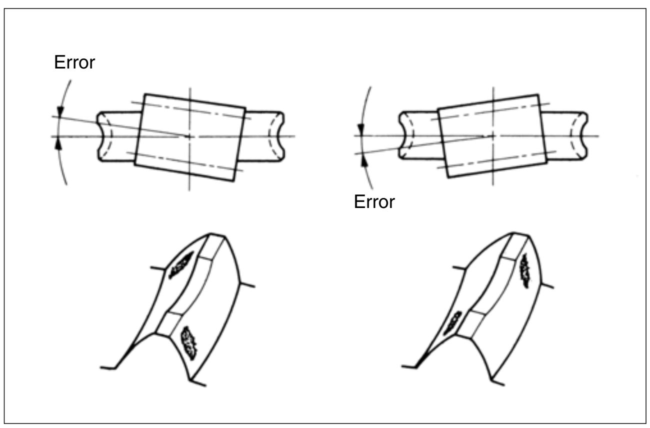


Fig. 6.6 Poor tooth contact due to shaft angle error

(2) Center Distance Error

Even when exaggerated center distance errors exist, as shown in Figure 6.7, the results are crossed contact. Such errors not only cause bad contact but also greatly influence backlash.
A positive center distance error causes increased backlash. A negative error will decrease backlash and may result in a tight mesh, or even make it impossible to assemble.

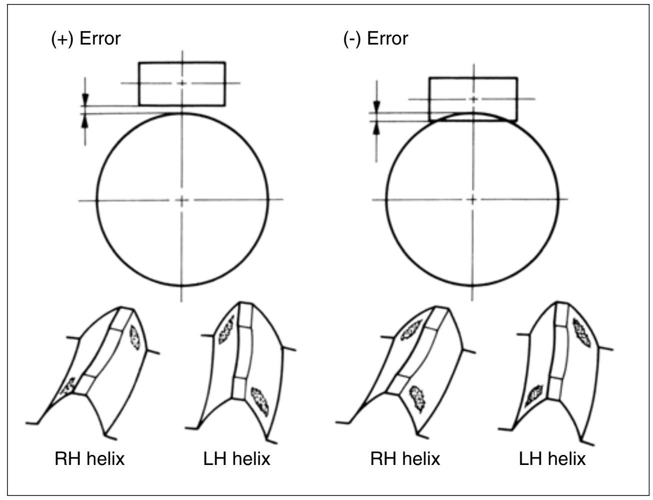


Fig.6.7 Poor tooth contact due to center distance error

(3) Locating Distance Error

Figure 6.8 shows the resulting poor contact from locating distance error of the worm wheel. From the figure, we can see the contact shifts toward the worm wheel tooth's edge. The direction of shift in the contact area matches the direction of worm wheel locating error. This error affects backlash, which tends to decrease as the error increases. The error can be diminished by micro-adjustment of the worm wheel in the axial direction.

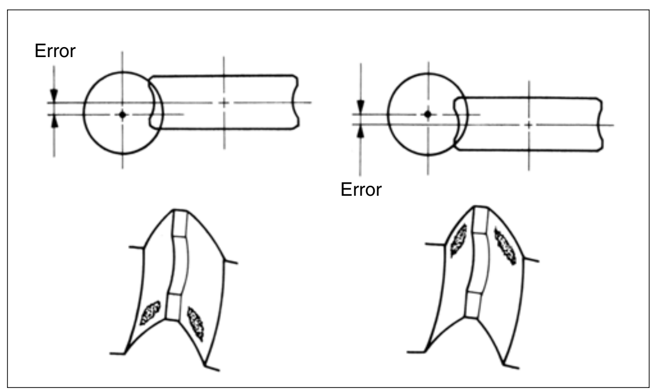


Fig.6.8 Poor tooth contact due to mounting distance error

7 LUBRICATION OF GEARS

The purpose of lubricating gears is as follows:

1. Promote sliding between teeth to reduce the coefficient of friction μ .
2. Limit the temperature rise caused by rolling and sliding friction.

To avoid difficulties such as tooth wear and premature failure, the correct lubricant must be chosen.

7.1 Methods of Lubrication

There are three gear lubrication methods in general use:

(1) Grease lubrication.

(2) Splash lubrication (oil bath method).

(3) Forced oil circulation lubrication.

There is no single best lubricant and method. Choice depends upon tangential speed (m/s) and rotaing speed (min^{-1}) .

At low speed, grease lubrication is a good choice. For medium and high speeds, splash lubrication and forced oil circulation lubrication are more appropriate, but there are exceptions. Sometimes, for maintenance reasons, a grease lubricant is used even with high speed.

Table 7.1 presents lubricants, methods and their applicable ranges of speed.

Table 7.1- ① Ranges of tangential speed (m/s) for spur and bevel gears

No.	Lubrication	Range of tangential speed v (m/s) 0 5 10 15 20 25
1	Grease lubrication	←→
2	Splash lubrication	←→
3	Forced oil circultion lubrication	←

Table 7.1 − ②Ranges of sliding speed (m/s) for worm wheels

No.	Lubrication	Range of sliding speed v_s (m/s) 0 5 10 15 20 25
1	Grease lubrication	←→
2	Splash lubrication	←→
3	Forced oil circultion lubrication	←

The following is a brief discussion of the three lubrication methods.

(1) Grease Lubrication

Grease lubrication is suitable for any gear system that is open or enclosed, so long as it runs at low speed. There are three major points regarding grease:

⊙ Choosing a lubricant with suitable cone penetration.

A lubricant with good fluidity is especially effective in an enclosed system.

⊙ Not suitable for use under high load and continuous operation.

The cooling effect of grease is not as good as lubricating oil. So it may become a problem with temperature rise under high load and continuous operating conditions.

⊙ Proper quantity of grease

There must be sufficient grease to do the job. However, too much grease can be harmful, particularly in an enclosed system. Excess grease will cause agitation, viscous drag and result in power loss.

(2) Splash Lubrication (Oil Bath Method)

Splash lubrication is used with an enclosed system. The rotating gears splash lubricant onto the gear system and bearings. It needs at least 3 m/s tangential speed to be effective. However, splash lubrication has several problems, two of them being oil level and temperature limitation.

① Oil level

There will be excess agitation loss if the oil level is too high. On the other hand, there will not be effective lubrication or ability to cool the gears if the level is too low. Table 7.2 shows guide lines for proper oil level.
Also, the oil level during operation must be monitored, as contrasted with the static level, in that the oil level will drop when the gears are in motion. This problem may be countered by raising the static level of lubrilling an oil pan.

② Temperature limitation.

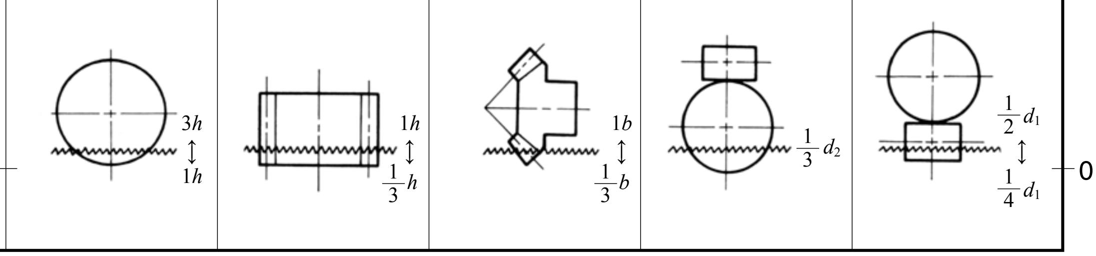
The temperature of a gear system may rise because of friction loss due to gears, bearings and lubricant agitation. Rising temperature may cause one or more of the following problems:

- Lower viscosity of lubricant
- Accelerated degradation of lubricant.
- Deformation of housing, gears and shafts.
- Decreased backlash.

New high-performance lubricants can withstand up to 80°C to 90°C.
This temperature can be regarded as the limit. If the lubricant's temperature is expected to exceed this limit, cooling fins should be added to the gear box, or a cooling fan incorporated into the system.

Table 7.2 Adequate oil level

Type of	Spur gears and helical gears		Bevel gears	Worm gear pair	
Gear	Horizontal shaft	Vertical shaft	(Horizontal shaft)	Worm - above	Worm -below
Oil level Level 0					



h = Tooth depth, b = Facewidth, d_2 = Reference diameter of worm wheel, d_1 = Reference diameter of worm

(3) Forced Oil Circulation Lubrication

Forced oil circulation lubrication applies lubricant to the contact portion of the teeth by means of an oil pump. There are drop, spray and oil mist methods of application.

- Drop Method

  An oil pump is used to suck-up the lubricant and then directly drop it on the contact portion of the gears via a delivery pipe.

- Spray Method

  An oil pump is used to spray the lubricant directly on the contact area of the gears.

- Oil Mist Method

  Lubricant is mixed with compressed air to form an oil mist that is sprayed against the contact region of the gears. It is especially suitable for high-speed gearing.

Oil tank, pump, filter, piping and other devices are needed in the forced oil lubrication system. Therefore, it is used only for special high-speed or large gear box applications.
By filtering and cooling the circulating lubricant, the right viscosity and cleanliness can be maintained. This is considered to be the best way to lubricate gears.

7.2 Gear Lubricants

An oil film must be formed at the contact surface of the teeth to minimize friction and to prevent dry metal-to-metal contact.

The lubricant should have the properties listed in Table 7.3.

Table 7.3 The properties that lubricant should possess

No.	Properties	Description
1	Correct and proper viscosity	Lubricant should maintain proper vicosity to form a stable oil film at the specified temperature and speed of operation.
2	Antiscoring property	Lubricant should have the property to prevent the scoring failure of tooth surface while under high-pressure of load.
3	Oxidization and heat stability	A good lubricant should not oxidize easily and must perform in moist and high-temperature environment for long duration.
4	Water antiaffinity property	Moisture tends to condense due to temperature change when the gears are stopped. The lubricant should have the property of isolating moisture and water from lubricant
5	Antifoam property	If the lubricant foams under agitation, it will not provide a good oil film. Antifoam property is a vital requirement.
6	Anticorrosion property	Lubrication should be neutral and stable to prevent corrosion from rust that may mix into the oil.

(1) Viscosity of Lubricant

The correct viscosity is the most important consideration in choosing a proper lubricant. The viscosity grade of industrial lubricant is regulated in JIS K 2001. Table 7.4 expresses ISO viscosity grade of industrial lubricants.

Besides ISO viscosity classifications, Table 7.5 contains AGMA viscosity grades and their equivalent ISO viscosity grades.

Table 7.4 ISO viscosity grade of industrial lubricant (JIS K 2001)

ISO Viscosity grade	Kinematic viscosity center value $10^{-6}m^2/s$(cSt) (40°C)	Kinematic viscosity range $10^{-6}m^2/s$(cSt) (40°C)
ISO VG 2	2.2	More than 1.98 and less than 2.42
ISO VG 3	3.2	More than 2.88 and less than 3.52
ISO VG 5	4.6	More than 4.14 and less than 5.06
ISO VG 7	6.8	More than 6.12 and less than 7.48
ISO VG 10	10	More than 9.0 and less than 11.0
ISO VG 15	15	More than 13.5 and less than 16.5
ISO VG 22	22	More than 19.8 and less than 24.2
ISO VG 32	32	More than 28.8 and less than 35.2
ISO VG 46	46	More than 41.4 and less than 50.6
ISO VG 68	68	More than 61.2 and less than 74.8
ISO VG 100	100	More than 90.0 and less than 110
ISO VG 150	150	More than 135 and less than 165
ISO VG 220	220	More than 198 and less than 242
ISO VG 320	320	More than 288 and less than 352
ISO VG 460	460	More than 414 and less than 506
ISO VG 680	680	More than 612 and less than 748
ISO VG 1000	1000	More than 900 and less than 1100
ISO VG 1500	1500	More than 1350 and less than 1650

Table 7.5 AGMA viscosity grades

AGMA No.of gear oil		ISO viscosity grades
R & O type	EP type	
1		VG 46
2	2 EP	VG 68
3	3 EP	VG 100
4	4 EP	VG 150
5	5 EP	VG 220
6	6 EP	VG 320
7 7 comp	7 EP	VG 460
8 8 comp	8 EP	VG 680
8 Acomp		VG 1000
9	9 EP	VG 1500

(2) Selection of Lubricant

It is practical to select a lubricant by following the catalog or technical manual of the manufacturer. Table 7.6 is the application guide from AGMA 250.03 "Lubrication of Industrial Enclosed Gear Drives".

Table 7.6 Recommended lubricants by AGMA

Gear type		Size of gear equipment (mm)		AGMA No. Ambient temperature °C −10 ~ 16	10 ~ 52
Parallel shaft system	Single stage reduction	Center distance (Output side)	Less than 200	2 − 3	3 − 4
			200 ~500	2 − 3	4 − 5
			More than 500	3 − 4	4 − 5
	Double stage reduction		Less than 200	2 − 3	3 − 4
			200 ~500	3 − 4	4 − 5
			More thn 500	3 − 4	4 − 5
	Triple stage reduction		Less than 200	2 − 3	3 − 4
			200 ~500	3 − 4	4 − 5
			More than 500	4 − 5	5 − 6
Planetary gear system		O.D. of gear casing	Less than 400	2 − 3	3 − 4
			More than 400	3 − 4	4 − 5
Straight and spiral bevel gearing		Cone distance	Less than 300	2 − 3	4 − 5
			More than 300	3 − 4	5 − 6
Gearmotor				2 − 3	4 − 5
High Speed Gear Equipment				1	2

Table 7.7 is the application guide chart for worm gear pair from AGMA 250.03.

Table 7.7 Recommended lubricants for worm gear pair by AGMA

Type of worm	Center distance mm	Rotating speed of worm min^{-1}	Ambient temperature, °C −10 ~ 16	10 ~ 52	Rotating speed of Worm min^{-1}	Ambient temperature, °C −10 ~ 16	10 ~ 52
Cylindrical type	≤ 150	≤700	7 Comp	8 Comp	700 <	7 Comp	8 Comp
	150 ~ 300	≤450			450 <		
	300 ~ 460	≤300			300 <		
	460 ~ 600	≤250			250 <		
	600 <	≤200			200 <		
Enveloping type	≤ 150	≤700	8 Comp	8 AComp	700 <	8 Comp	
	150 ~ 300	≤450			450 <		
	300 ~ 460	≤300			300 <		
	460 ~ 600	≤250			250 <		
	600 <	≤200			200 <		

Table 7.8 expresses the reference value of viscosity of lubricant used in the equations for the strength of worm gears in JGMA 405-01.

Table 7.8 Reference values of viscosity unit: cSt/37.8ºC

Operating temperature		Sliding speed m/s		
Maximum running	Starting temperature	Less than 2.5	2.5 through 5	More than 5
0ºC ~ 10ºC	−10ºC ~ 0ºC	110 ~ 130	110 ~ 130	110 ~ 130
	More than 0ºC	110 ~ 150	110 ~ 150	110 ~ 150
10ºC ~ 30ºC	More than 0ºC	200 ~ 245	150 ~ 200	150 ~ 200
30ºC ~ 55ºC	More than 0ºC	350 ~ 510	245 ~ 350	200 ~ 245
55ºC ~ 80ºC	More than 0ºC	510 ~ 780	350 ~ 510	245 ~ 350
80ºC ~ 100ºC	More than 0ºC	900 ~ 1100	510 ~ 780	350 ~ 510

After making decision about which grade of viscosity to select, taking into consideration the usage (for spur gear, worm gear pair etc.) and usage conditions (dimensions of mechanical equipment, ambient temperature etc.), choose the appropriate lubricant. Technical manual of the libricant manufacturer may be of great help.

8 GEAR FORCES

In designing a gear, it is important to analyze the magnitude and direction of the forces acting upon the gear teeth, shafts, bearings, etc. In analyzing these forces, an idealized assumption is made that the tooth forces are acting upon the central part of the tooth flank.

Table 8.1 presents the equations for tangential (circumferential) force F_t (kgf), axial (thrust) force F_x(kgf), and radial force F_r in relation to the transmission force F_n acting upon the central part of the tooh flank.
T and T_1 shown therein represent input torque (kgf·m).

Table 8.1 Forces acting upon a gear

Types of gears		F_t:Tangential force	F_x : Axial force	F_r : Radial force
Spur gear		$F_t = \frac{2000T}{d}$	――――	$F_t \tan\alpha$
Helical gear			$F_t \tan\beta$	$F_t \frac{\tan\alpha_n}{\cos\beta}$
Straight bevel gear		$F_t = \frac{2000T}{d_m}$	$F_t \tan\alpha \sin\delta$	$F_t \tan\alpha \cos\delta$
Spiral bevel gear		d_m is the central reference diameter $d_m = d - b \sin\delta$	When convex surface is working:	
			$\frac{F_t}{\cos\beta_m}(\tan\alpha_n \sin\delta - \sin\beta_m \cos\delta)$	$\frac{F_t}{\cos\beta_m}(\tan\alpha_n \cos\delta + \sin\beta_m \sin\delta)$
			When concave surface is working:	
			$\frac{F_t}{\cos\beta_m}(\tan\alpha_n \sin\delta + \sin\beta_m \cos\delta)$	$\frac{F_t}{\cos\beta_m}(\tan\alpha_n \cos\delta - \sin\beta_m \sin\delta)$
Worm gear pair	Worm (Driver)	$F_t = \frac{2000T_1}{d_1}$	$F_t \frac{\cos\alpha_n \cos\gamma - \mu \sin\gamma}{\cos\alpha_n \sin\gamma + \mu \cos\gamma}$	$F_t \frac{\sin\alpha_n}{\cos\alpha_n \sin\gamma + \mu \cos\gamma}$
	Worm Wheel (Driven)	$F_t \frac{\cos\alpha_n \cos\gamma - \mu \sin\gamma}{\cos\alpha_n \sin\gamma + \mu \cos\gamma}$	F_t	
Screw gear $\left(\begin{array}{l}\Sigma = 90° \\ \beta = 45°\end{array}\right)$	Driver gear	$F_t = \frac{2000T_1}{d_1}$	$F_t \frac{\cos\alpha_n \sin\beta - \mu \cos\beta}{\cos\alpha_n \cos\beta + \mu \sin\beta}$	$F_t \frac{\sin\alpha_n}{\cos\alpha_n \cos\beta + \mu \sin\beta}$
	Driven gear	$F_t \frac{\cos\alpha_n \sin\beta - \mu \cos\beta}{\cos\alpha_n \cos\beta + \mu \sin\beta}$	F_t	

8.1 Forces in a Spur Gear Mesh

The Spur Gear's transmission force F_n, which is normal to the tooth surface , as in Figure 8.1, can be resolved into a tangential component, F_t , and a radial component, F_r. Refer to Equation (8.1).

$$\left.\begin{array}{l} F_t = F_n \cos\alpha' \\ F_r = F_n \sin\alpha' \end{array}\right\} \qquad (8.1)$$

There will be no axial force, F_x.

The direction of the forces acting on the gears are shown in Figure 8.2. The tangential component of the drive gear, F_{t1} is equal to the driven gear's tangential component, F_{t2}, but the directions are opposite. Similarly, the same is true of the radial components.

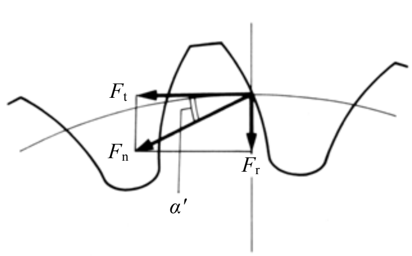


Fig.8.1 Forces acting on a spur gear mesh

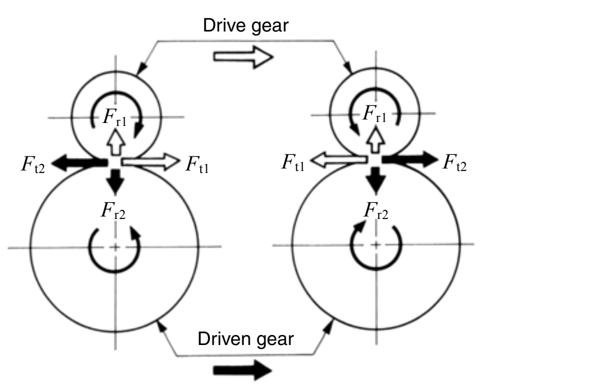


Fig.8.2 Directions of forces acting on a spur gear mesh

8.2 Forces in a Helical Gear Mesh

The helical gear's transmission force, F_n, which is normal to the tooth surface, can be resolved into a tangential component, F_1, and a radial component, F_r, as shown in Figure 8.3.

$$\left.\begin{aligned} F_1 &= F_n \cos\alpha_n \\ F_r &= F_n \sin\alpha_n \end{aligned}\right\} \qquad (8.2)$$

The tangential component, F_1, can be further resolved into circular subcomponent, F_t, and axial thrust subcomponent, F_x.

$$\left.\begin{aligned} F_t &= F_1 \cos\beta \\ F_x &= F_1 \sin\beta \end{aligned}\right\} \qquad (8.3)$$

Substituting and manipulating the above equations result in:

$$\left.\begin{aligned} F_x &= F_t \tan\beta \\ F_r &= F_t \frac{\tan\alpha_n}{\cos\beta} \end{aligned}\right\} \qquad (8.4)$$

The directions of forces acting on a helical gear mesh are shown in Figure 8.4.
The axial thrust sub-component from drive gear, F_{x1}, equals the driven gear's, F_{x2}, but their directions are opposite.
Again, this case is the same as tangential components and radial components.

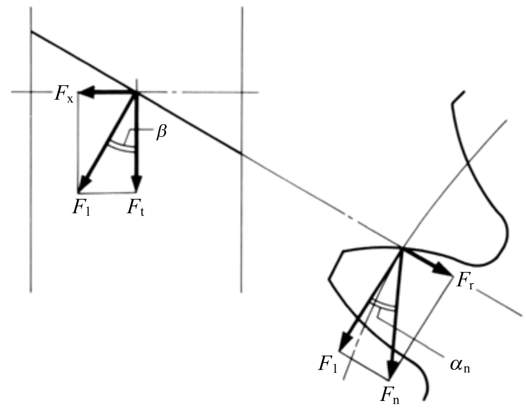


Fig.8.3 Forces acting on a helical gear mesh

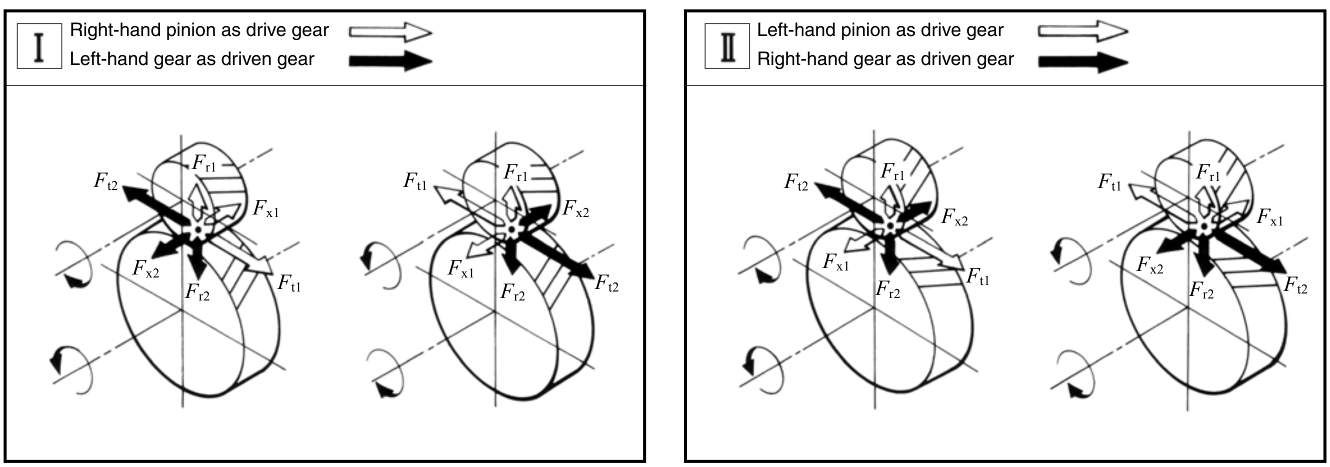


Fig.8.4 Directions of forces acting on a helical gear mesh

8.3 Forces in a Straight Bevel Gear Mesh

The forces acting on a straight bevel gear are shown in Figure 8.5. The force which is normal to the central part of the tooth face, F_n, can be split into tangential component, F_t, and radial component, F_1, in the normal plane of the tooth.

$$\left.\begin{aligned} F_t &= F_n \cos\alpha_n \\ F_1 &= F_n \sin\alpha_n \end{aligned}\right\} \qquad (8.5)$$

Again, the radial component, F_1, can be divided into an axial force, F_x, and a radial force, F_r, perpendicular to the axis.

$$\left.\begin{aligned} F_x &= F_1 \sin\delta \\ F_r &= F_1 \cos\delta \end{aligned}\right\} \qquad (8.6)$$

And the following can be derived:

$$\left.\begin{aligned} F_x &= F_t \tan\alpha_n \sin\delta \\ F_r &= F_t \tan\alpha_n \cos\delta \end{aligned}\right\} \qquad (8.7)$$

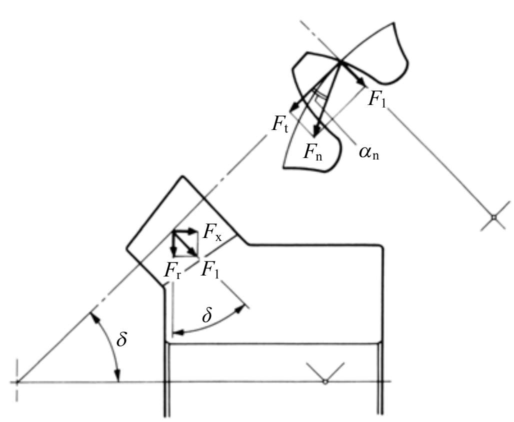


Fig.8.5 Forces acting on a straight bevel gear mesh

Let a pair of straight bevel gears with a shaft angle $\Sigma = 90°$, a pressure angle $\alpha_n = 20°$ and tangential force, F_t, to the central part of tooth face be 100. Axial force, F_x, and radial force, F_r, will be as presented in Table 8.2.

Table 8.2 $\frac{\text{Axial force } F_x}{\text{Radial force } F_r}$ Values

(1) Pinion

Forces on the gear tooth	Gear ratio z_2/z_1						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Axial force / Radial force	25.7 / 25.7	20.2 / 30.3	16.3 / 32.6	13.5 / 33.8	11.5 / 34.5	8.8 / 35.3	7.1 / 35.7

(2) Gear

Forces on the gear tooth	Gear ratio z_2/z_1						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Axial force / Radial force	25.7 / 25.7	30.3 / 20.2	32.6 / 16.3	33.8 / 13.5	34.5 / 11.5	35.3 / 8.8	35.7 / 7.1

Figure 8.6 contains the directions of forces acting on a straight bevel gear mesh. In the meshing of a pair of straight bevel gears with shaft angle $\Sigma = 90°$, the axial force acting on drive gear F_{x1} equals the radial force acting on driven gear F_{r2}. Similarly, the radial force acting on drive gear F_{r1} equals the axial force acting on driven gear F_{x2}. The tangential force F_{t1} equals that of F_{t2}.

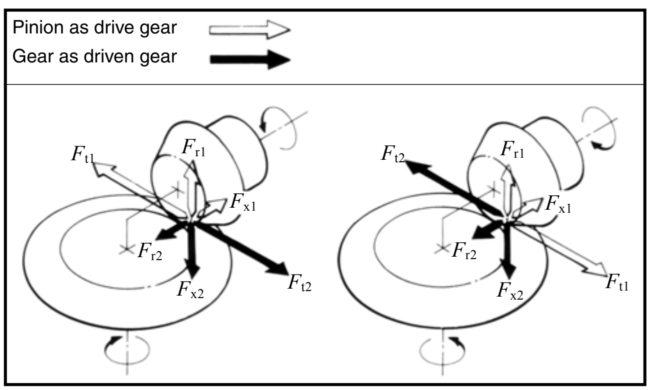


Fig.8.6 Directions of forces acting on a straight bevel gear mesh

All the forces have relations as per Equations (8.8).

$$\left.\begin{aligned} F_{t1} &= F_{t2} \\ F_{r1} &= F_{x2} \\ F_{x1} &= F_{r2} \end{aligned}\right\} \qquad (8.8)$$

8.4 Forces in A Spiral Bevel Gear Mesh

Spiral bevel gear teeth have convex and concave sides. Depending on which surface the force is acting on, the direction and magnitude changes. They differ depending upon which is the driver and which is the driven.

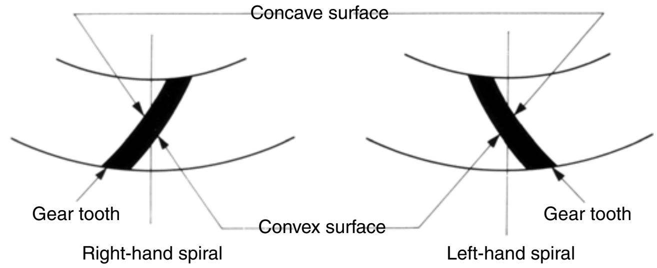


Fig.8.7 Convex surface and concave surface of a spiral bevel gear

Figure 8.7 presents the profile orientations of right-hand and left-hand spiral teeth. If the profile of the driving gear is convex, then the profile of the driven gear must be concave. Table 8.3 presents the convex/concave relationships.

Table 8.3 Concave and convex sides of a spiral bevel gear
Right-hand gear as drive gear

Rotational direction of drive gear	Meshing tooth face	
	Right-hand drive gear	Left-hand driven gear
Clockwise	Convex	Concave
Counterclockwise	Concave	Convex

Left-hand gear as drive gear

Rotational direction of drive gear	Meshing tooth face	
	Left-hand drive gear	Right-hand driven gear
Clockwise	Concave	Convex
Counterclockwise	Convex	Concave

(1) Forces on Convex Side Profile

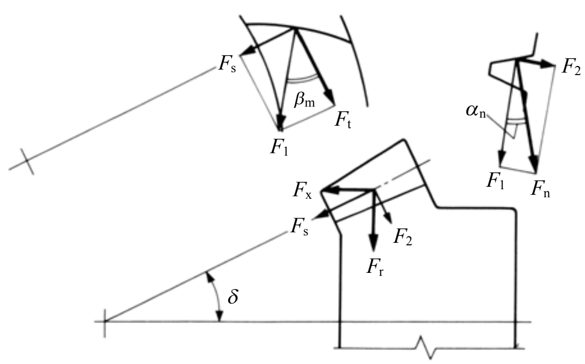


Fig.8.8 When meshing on the convex side of tooth face

The transmission force, F_n, can be resolved into components F_1 and F_2. (See Figure 8.8).

$$\left.\begin{aligned} F_1 &= F_n \cos\alpha_n \\ F_2 &= F_n \sin\alpha_n \end{aligned}\right\} \tag{8.9}$$

Then F_1 can be resolved into components F_t and F_s:

$$\left.\begin{aligned} F_t &= F_1 \cos\beta_m \\ F_s &= F_1 \sin\beta_m \end{aligned}\right\} \tag{8.10}$$

On the axial surface, F_2 and F_s can be resolved into axial and radial subcomponents.

$$\left.\begin{aligned} F_x &= F_2 \sin\delta - F_s \cos\delta \\ F_r &= F_2 \cos\delta + F_s \sin\delta \end{aligned}\right\} \tag{8.11}$$

By substitution and manipulation, we obtain:

$$\left.\begin{aligned} F_x &= \frac{F_t}{\cos\beta_m}(\tan\alpha_n \sin\delta - \sin\beta_m \cos\delta) \\ F_r &= \frac{F_t}{\cos\beta_m}(\tan\alpha_n \cos\delta + \sin\beta_m \sin\delta) \end{aligned}\right\} \tag{8.12}$$

(2) Forces on a Concave Side Profile

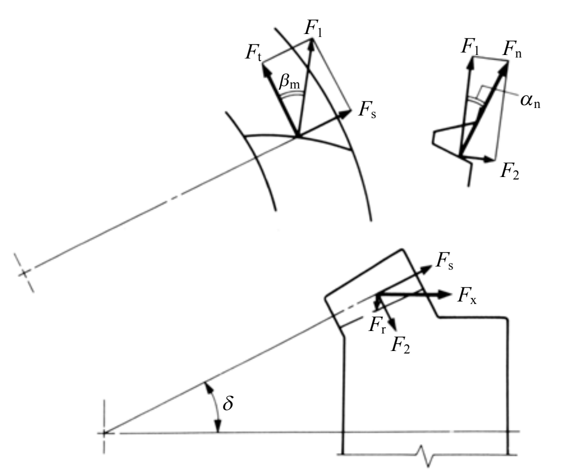


Fig.8.9 When meshing on the concave side of tooth face

On the surface which is normal to the tooth profile at the central portion of the tooth, the transmission force F_n can be split into F_1 and F_2. See Figure 8.9:

$$\left.\begin{aligned} F_1 &= F_n \cos\alpha_n \\ F_2 &= F_n \sin\alpha_n \end{aligned}\right\} \tag{8.13}$$

And F_1 can be separated into components F_t and F_s on the pitch surface:

$$\left.\begin{aligned} F_t &= F_1 \cos\beta_m \\ F_s &= F_1 \sin\beta_m \end{aligned}\right\} \tag{8.14}$$

So far, the equations are identical to the convex case. However, differences exist in the signs for equation terms. On the axial surface, F_2 and F_s can be resolved into axial and radial subcomponents. Note the sign differences.

$$\left.\begin{aligned} F_x &= F_2 \sin\delta + F_s \cos\delta \\ F_r &= F_2 \cos\delta - F_s \sin\delta \end{aligned}\right\} \tag{8.15}$$

The above can be manipulated to yield:

$$\left.\begin{aligned} F_x &= \frac{F_t}{\cos\beta_m}(\tan\alpha_n \sin\delta + \sin\beta_m \cos\delta) \\ F_r &= \frac{F_t}{\cos\beta_m}(\tan\alpha_n \cos\delta - \sin\beta_m \sin\delta) \end{aligned}\right\} \tag{8.16}$$

Let a pair of spiral bevel gears have a shaft angle $\Sigma = 90°$, a pressure angle $\alpha_n = 20°$, and a spiral angle $\beta_m = 35°$. If the tangential force, F_t to the central portion of the tooth face is 100, the axial force, F_x, and radial force, F_r, have the relationship shown in Table 8.4.

Table 8.4 Values of $\dfrac{\text{Axial force, } F_x}{\text{Radial force, } F_r}$

(1) Pinion

Meshing tooth face	Gear ratio z_2/z_1						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Concave side of tooth	$\frac{80.9}{-18.1}$	$\frac{82.9}{-1.9}$	$\frac{82.5}{8.4}$	$\frac{81.5}{15.2}$	$\frac{80.5}{20.0}$	$\frac{78.7}{26.1}$	$\frac{77.4}{29.8}$
Convex side of tooth	$\frac{-18.1}{80.9}$	$\frac{-33.6}{75.8}$	$\frac{-42.8}{71.1}$	$\frac{-48.5}{67.3}$	$\frac{-52.4}{64.3}$	$\frac{-57.2}{60.1}$	$\frac{-59.9}{57.3}$

(2) Gear

Meshing tooth face	Gear ratio z_2/z_1						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Concave side of tooth	$\frac{80.9}{-18.1}$	$\frac{75.8}{-33.6}$	$\frac{71.1}{-42.8}$	$\frac{67.3}{-48.5}$	$\frac{64.3}{-52.4}$	$\frac{60.1}{-57.2}$	$\frac{57.3}{-59.9}$
Convex side of tooth	$\frac{-18.1}{80.9}$	$\frac{-1.9}{82.9}$	$\frac{8.4}{82.5}$	$\frac{15.2}{81.5}$	$\frac{20.0}{80.5}$	$\frac{26.1}{78.7}$	$\frac{29.8}{77.4}$

The value of axial force, F_x, of a spira bevel gear, from Table 8.4, could become negative. At that point, there are forces tending to push the two gears together. If there is any axial play in the bearing, it may lead to the undesirable condition of the mesh having no backlash. Therefore, it is important to pay particular attention to axial plays.

From Table 8.4(2), we understand that axial turning point of axial force, F_x, changes from positive to negative in the range of gear ratio from 1.5 to 2.0 when a gear carries force on the convex suide. The precise turning point of axial force, F_x, is at the gear ratio z_2/z_1 =1.57357.

Figure 8.10 describes the forces for a pair of spiral bevel gears with shaft angle $\Sigma = 90°$, pressure angle $\alpha_n = 20°$, spiral angle $\beta_m = 35°$ and the gear ratio z_2/z_1, ranging from 1 to 1.57357. Figure 8.11 expresses the forces of another pair of spiral bevel gears taken with the gear ratio z_2/z_1 equal to or larger than 1.57357.

$\Sigma = 90°$, $\alpha_n = 20°$, $\beta_m = 35°$, $u < 1.57357$

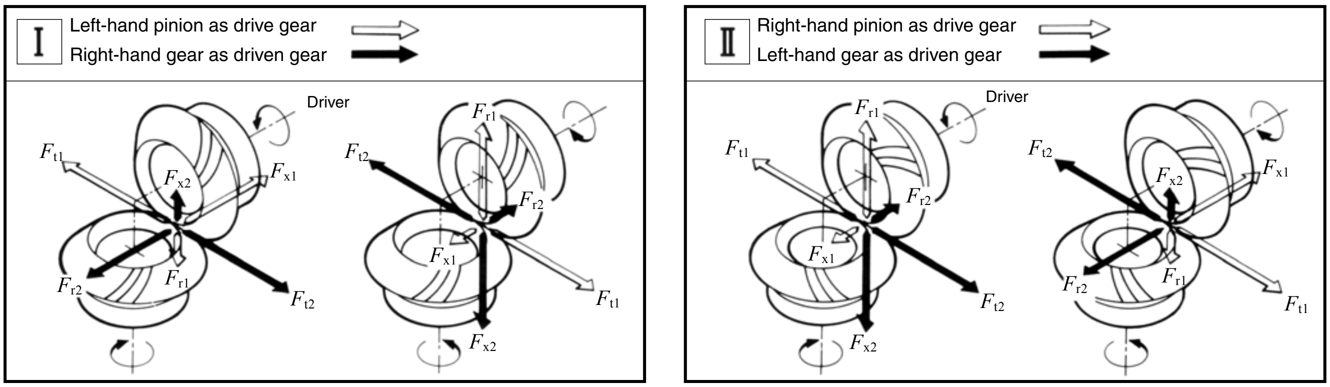


Fig.8.10 The direction of forces carried by spiral bevel gears (1)

$\Sigma = 90°$, $\alpha_n = 20°$, $\beta_m = 35°$, $u \geq 1.57357$

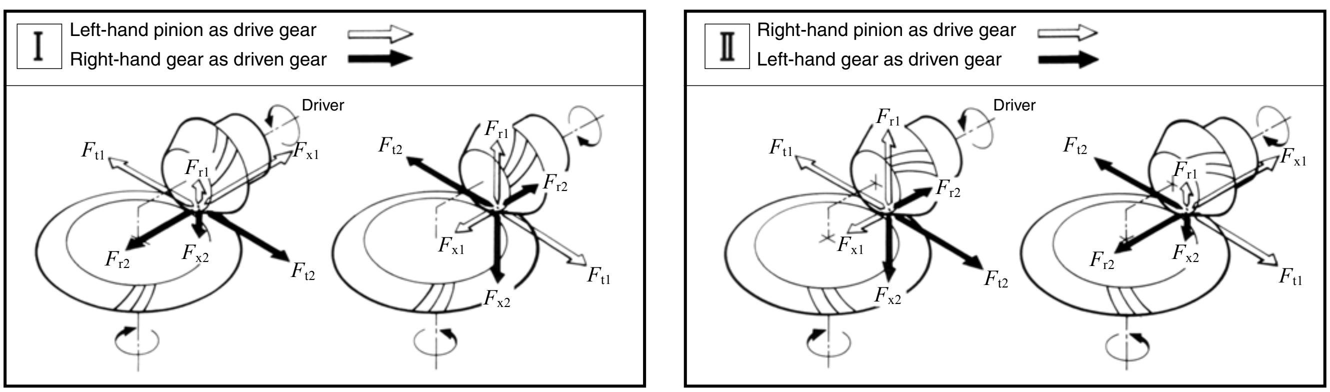


Fig.8.11 The direction of forces carried by spiral bevelgears (2)

8.5 Forces in a Worm Gear Pair Mesh

(1) Worm as the Driver

For the case of a worm as the driver, Figure 8.12, the transmission force, F_n, which is normal to the tooth surface at the pitch circle can be resolved into components F_1 and F_{r1}.

$$\left.\begin{aligned} F_1 &= F_n \cos\alpha_n \\ F_{r1} &= F_n \sin\alpha_n \end{aligned}\right\} \qquad (8.17)$$

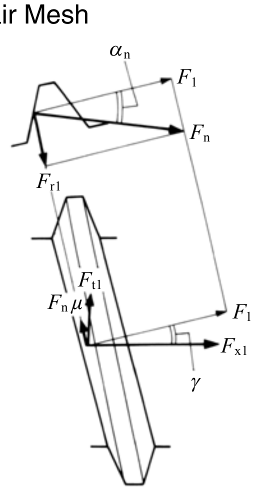


Fig..12 Forces acting on the tooth surface of a worm

At the pitch surface of the worm, there is, in addition to the tangential component, F_1, a friction sliding force on the tooth surface, $F_n\mu$. These two forces can be resolved into the circular and axial directions as:

$$\left.\begin{aligned} F_{t1} &= F_1 \sin\gamma + F_n\mu \cos\gamma \\ F_{x1} &= F_1 \cos\gamma - F_n\mu \sin\gamma \end{aligned}\right\} \qquad (8.18)$$

and by substitution, the result is:

$$\left.\begin{aligned} F_{t1} &= F_n(\cos\alpha_n \sin\gamma + \mu \cos\gamma) \\ F_{x1} &= F_n(\cos\alpha_n \cos\gamma - \mu \sin\gamma) \\ F_{r1} &= F_n \sin\alpha_n \end{aligned}\right\} \qquad (8.19)$$

Figure 8.13 presents the direction of forces in a worm gear pair mesh with a shaft angle $\Sigma = 90°$. These forces relate as follows:

$$\left.\begin{aligned} F_{x1} &= F_{t2} \\ F_{t1} &= F_{x2} \\ F_{r1} &= F_{r2} \end{aligned}\right\} \qquad (8.20)$$

In a worm gear pair mesh with a shaft angle $\Sigma = 90°$, the axial force acting on drive gear F_{x1} equals the tangential force acting on driven gear F_{t2}. Similarly, the tangential force acting on drive gear F_{t1} equals the axial force acting on driven gear F_{x2}. The radial force F_{r1} equals that of F_{r2}.

The equations concerning worm and worm wheel forces contain the coefficient μ. The coefficient of friction has a great effect on the transmission of a worm gear pair. Equation (8-21) presents the efficiency when the worm is the driver.

$$\left.\begin{aligned} \eta_R &= \frac{T_2}{T_1 i} = \frac{F_{t2}}{F_{t1}}\tan\gamma \\ &= \frac{\cos\alpha_n \cos\gamma - \mu \sin\gamma}{\cos\alpha_n \sin\gamma + \mu \cos\gamma}\tan\gamma \end{aligned}\right\} \qquad (8.21)$$

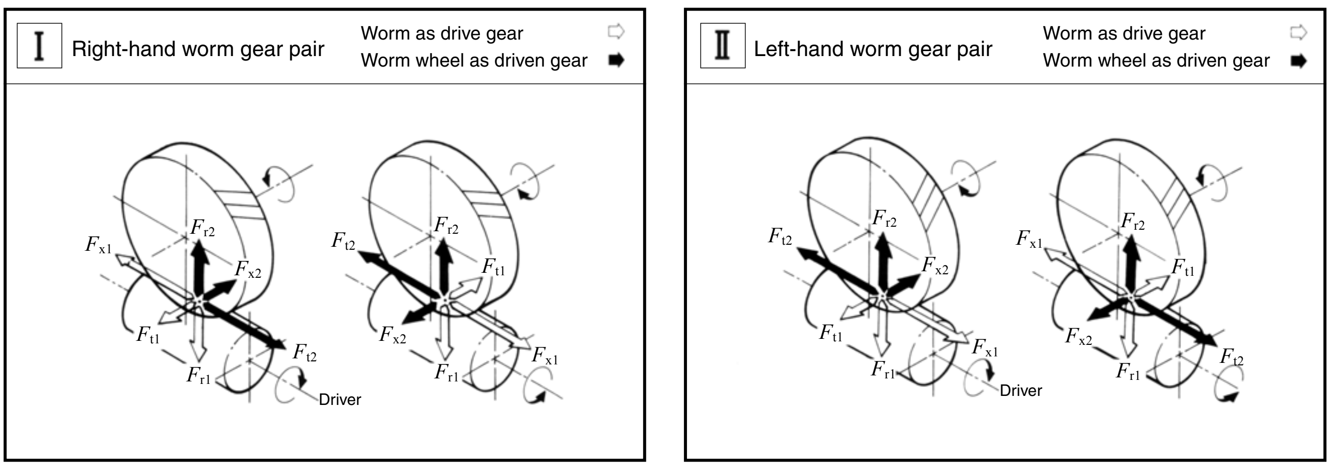


Figure 8.13 Direction of forces in a worm gear pair mesh

(2) Worm Wheel as the Driver

For the case of a worm wheel as the driver, the forces are as in Figure 8.14 and per Equations (8.22).

$$\left.\begin{aligned} F_{t2} &= F_n(\cos\alpha_n \cos\gamma + \mu \sin\gamma) \\ F_{x2} &= F_n(\cos\alpha_n \sin\gamma - \mu \cos\gamma) \\ F_{r2} &= F_n \sin\alpha_n \end{aligned}\right\} \qquad (8.22)$$

When the worm and worm wheel are at 90° shaft angle, Equations (8.20) apply. Then, when the worm wheel is the driver, the transmission efficiency η_I is expressed as per Equation (8.23).

$$\left.\begin{aligned} \eta_I &= \frac{T_1 i}{T_2} = \frac{F_{t1}}{F_{t2}\tan\gamma} \\ &= \frac{\cos\alpha_n \sin\gamma - \mu\cos\gamma}{\cos\alpha_n \cos\gamma + \mu\sin\gamma}\,\frac{1}{\tan\gamma} \end{aligned}\right\} \qquad (8.23)$$

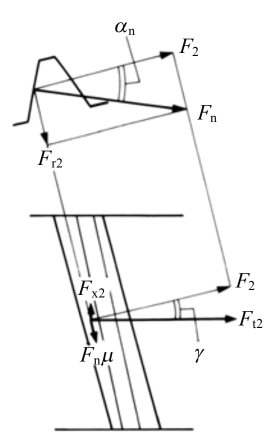


Fig.8.14 Forces in a worm gear pair mesh

8.6 Forces in a Screw Gear Mesh

The forces in a screw gear mesh are similar to those in a worm gear pair mesh. For screw gears that have a shaft angle $\Sigma = 90°$, merely replace the worm's lead angle γ, in Equation (8.22), with the screw gear's helix angle β_1.

In the general case when the shaft angle is not 90°, as in Figure 8.15, the driver screw gear has the same forces as for a worm mesh. These are expressed in Equations (8.24).

$$\left.\begin{aligned} F_{t1} &= F_n(\cos\alpha_n\cos\beta_1 + \mu\sin\beta_1) \\ F_{x1} &= F_n(\cos\alpha_n\sin\beta_1 - \mu\cos\beta_1) \\ F_{r1} &= F_n\sin\alpha_n \end{aligned}\right\} \tag{8.24}$$

Forces acting on the driven gear can be calculated per Equations (8.25).

$$\left.\begin{aligned} F_{t2} &= F_{x1}\sin\Sigma + F_{t1}\cos\Sigma \\ F_{x2} &= F_{t1}\sin\Sigma - F_{x1}\cos\Sigma \\ F_{r2} &= F_{r1} \end{aligned}\right\} \tag{8.25}$$

If the Σ term in Equation (8.25) is 90°, it becomes identical to Equation (8.20).
Figure 8.16 presents the direction of forces in a screw gear mesh when the shaft angle $\Sigma = 90°$, and $\beta_1 = \beta_2 = 45°$.

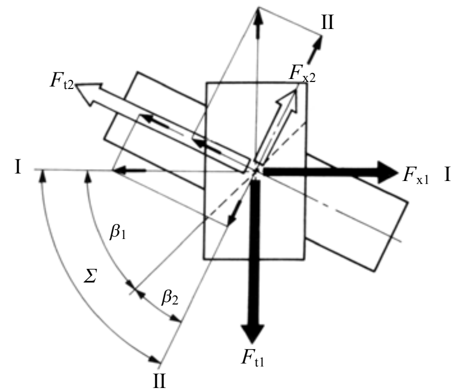


Fig.8.15 The forces in a screw gear mesh

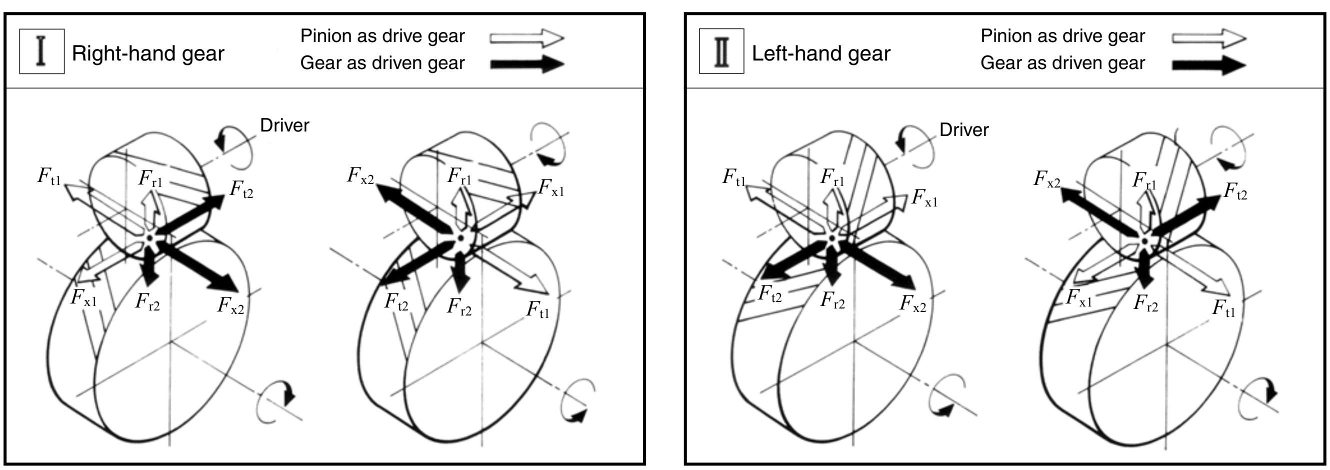


Fig.8.16 Direction of forces in a screw gear mesh

9 CONTACT RATIO

To assure continuous smooth tooth action, as one pair of teeth ceases action a succeeding pair of teeth must already have come into engagement. It is desirable to have as much overlap as is possible. A measure of this overlap action is the contact ratio.

When considering all types of gears, contact ratio is composed of :

Transverse contact ratio, ε_α
Overlap ratio, ε_β
Total contact ratio, ε_γ

9.1 Transverse Contact Ratio, ε_α

Transverse contact ratio (plane of rotation perpendicular to axes), ε_α is found by dividing the length of path of contact by the base pitch, p_b. There are three factors that influence the transverse contact ratio, ε_α. These are pressure angle, α', number of teeth z_1, z_2, and working tooth depth, h'.
In order to increase ε_α there are three means:

① Decrease the pressure angle.

Decreasing the pressure angle makes a longer line-of-action as it extends through the region between the two outside radii. Also, it is feasible to decrease the pressure angle by means of profile shifting.

② Increase the number of teeth.

As the number of teeth increases and pitch diameter grows, again there is a longer line-of-action in the region between the outside radii. For a fixed center distance, the transverse contact ratio will become bigger if the gear of smaller module with proportionately larger number of teeth, is used. (For instance, use SS1-30 in place of SS2-15).

③ Increase working depth.

Working depth h' of standard full depth tooth is twice as large as each module size. Therefore, increasing working depth requires a special tooth design, a "high-tooth".

1) Transverse Contact Ratio of Parallel Axes Gear

Table 9.1 presents equations of transverse contact ratio on parallel axes gear.

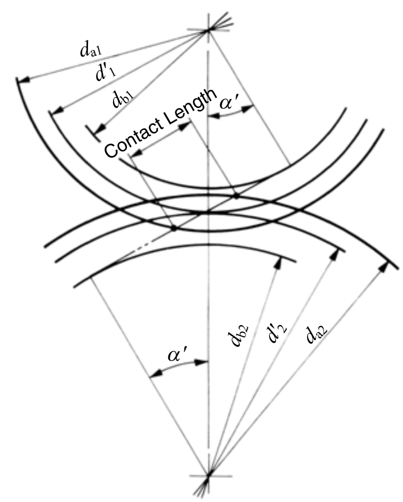


Fig.9.1 Transverse contact ratio ε_α

Table 9.1 Equations of transverse contact ratio on parallel axes gear, ε_α

No.	Type of gear mesh	Formula of transverse contact ratio, ε_α
1	Spur gear ① ②	$\dfrac{\sqrt{\left(\frac{d_{a1}}{2}\right)^2-\left(\frac{d_{b1}}{2}\right)^2}+\sqrt{\left(\frac{d_{a2}}{2}\right)^2-\left(\frac{d_{b2}}{2}\right)^2}-a\sin\alpha'}{\pi m\cos\alpha}$
2	Spur gear ① Rack ②	$\dfrac{\sqrt{\left(\frac{d_{a1}}{2}\right)^2-\left(\frac{d_{b1}}{2}\right)^2}+\frac{h_{a2}-x_1 m}{\sin\alpha}-\frac{d_1}{2}\sin\alpha}{\pi m\cos\alpha}$
3	Spur gear ① Internal gear ②	$\dfrac{\sqrt{\left(\frac{d_{a1}}{2}\right)^2-\left(\frac{d_{b1}}{2}\right)^2}-\sqrt{\left(\frac{d_{a2}}{2}\right)^2-\left(\frac{d_{b2}}{2}\right)^2}+a\sin\alpha'}{\pi m\cos\alpha}$
4	Helical gear ① ②	$\dfrac{\sqrt{\left(\frac{d_{a1}}{2}\right)^2-\left(\frac{d_{b1}}{2}\right)^2}+\sqrt{\left(\frac{d_{a2}}{2}\right)^2-\left(\frac{d_{b2}}{2}\right)^2}-a\sin\alpha'_t}{\pi m_t\cos\alpha_t}$

◎ An example of helical gear

$m_n = 3$ $\alpha_n = 20°$ $\beta = 30°$ $z_1 = 12$ $z_2 = 60$ $x_1 = +0.09809$ $x_2 = 0$
$a = 125$ $\alpha_t = 22.79588°$ $\alpha'_t = 23.1126°$ $m_t = 3.46410$
$d_{a1} = 48.153$ $d_{a2} = 213.842$ $d_{b1} = 38.322$ $d_{b2} = 191.611$
$\varepsilon_\alpha = 1.2939$

(2) Transverse Contact Ratio of Bevel Gears, ε_α

The transverse contact ratio of a bevel gear pair, ε_α can be derived from consideration of the equivalent spur gears, when viewed from the back cone. (See Figure 4.9 on page 404.)
Table 9.2 presents equations calculating the transverse contact ratio.

Table 9.2 Equations for transverse contact ratio for a bevel gear pair, ε_α

No.	Item	Symbol	Equations for Contact Ratio	
1	Back cone distance	R_v	$\frac{d}{2\cos\delta}$	
2	Base radius of an equivalent spur gear	R_{vb}	Straight bevel gear $R_v\cos\alpha$	Spiral bevel gear $R_v\cos\alpha_t$
3	Tip radius of an equivalent spur gear	R_{va}	$R_v + h_a$	
4	Transverse contact ratio	ε_α	Straight bevel gear $\frac{\sqrt{R_{va1}^2 - R_{vb1}^2} + \sqrt{R_{va2}^2 - R_{vb2}^2} - (R_{v1}+R_{v2})\sin\alpha}{\pi m\cos\alpha}$ Spiral bevel gear $\frac{\sqrt{R_{va1}^2 - R_{vb1}^2} + \sqrt{R_{va2}^2 - R_{vb2}^2} - (R_{v1}+R_{v2})\sin\alpha_t}{\pi m\cos\alpha_t}$	

◎ An example of spiral bevel gear
$m = 3$, $\alpha_n = 20°$, $\beta = 35°$, $z_1 = 20$, $z_2 = 40$, $\alpha_t = 23.95680°$
$d_1 = 60$, $d_2 = 120$, $R_{v1} = 33.54102$, $R_{v2} = 134.16408$
$R_{vb1} = 30.65152$, $R_{vb2} = 122.6061$, $h_{a1} = 3.4275$, $h_{a2} = 1.6725$
$R_{va1} = 36.9685$, $R_{va2} = 135.83658$
$\varepsilon_\alpha = 1.2825$

(3) Transverse Contact Ratio For Nonparallel and Nonintersecting Axes Gear Pairs, ε_α

Table 9.3 presents equations for contact ratio, ε_α, of nonparallel and nonintersecting gear meshes.
The equations are approximations by considering the worm gear pair mesh in the plane perpendicular to worm wheel axis and likening it to spur gear and rack mesh.

Table 9.3 Equations for transverse contact ratio of nonparallel and nonintersecting meshes, ε_α

No.	Type of gear mesh	Equation of transverse contact ratio, ε_α
1	Worm ① Worm wheel ②	$\frac{\frac{h_{a1}^2 - x_{t2}m_x}{\sin\alpha_x} + \sqrt{\left(\frac{d_t}{2}\right)^2 - \left(\frac{d_{b2}}{2}\right)^2} - \frac{d_2}{2}\sin\alpha_x}{\pi m_x\cos\alpha_x}$

◎ An example of worm gear pair mesh
$m_x = 3$, $\alpha_n = 20°$, $z_1 = 2$, $z_2 = 30$
$d_1 = 44$, $d_2 = 90$, $\gamma = 7.76517°$
$\alpha_x = 20.17024°$, $h_{a1} = 3$
$d_t = 96$, $d_{b2} = 84.48050$
$\varepsilon_\alpha = 1.8066$

9.2 Overlap Ratio, ε_β

Helical gears and spiral bevel gears have an overlap of tooth action in the axial direction. Overlap ratio is obtained by dividing gear width, b, by p_x, the axial pitch. (See the illustration in Figure 9.2.) Equations for calculating overlap ratio are presented in Table 9.4.

Table 9.4 Equation for overlap ratio, ε_β

No.	Type of Gear	Equation	Example
1	Helical gear	$\dfrac{b \sin\beta}{\pi m_n}$	$b = 50$, $\beta = 30°$, $m_n = 3$ $\varepsilon_\beta = 2.6525$
2	Spiral bevel gear	$\dfrac{R}{R-0.5b} \quad \dfrac{b \tan\beta_m}{\pi m}$	From Table 4.21 (Page 409): $R = 67.08204$, $b = 20$, $\beta_m = 35°$, $m = 3$ $\varepsilon_\beta = 1.7462$

NOTE: The module m in spiral bevel gear equation is the transverse module.

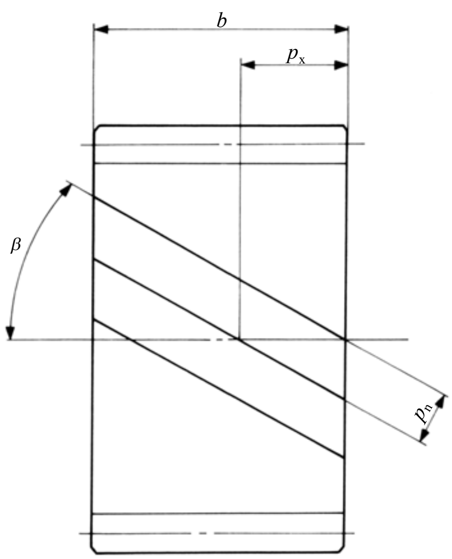


Fig.9.2 Overlap ratio, ε_β

10 GEAR NOISE

There are several causes of noise. The noise and vibration in rotating gears, especially at high loads and high speeds, need to be addressed. Following are ways to reduce the noise. These points should be considered in the design stage of gear systems.

(1) Use High-Precision Gears

- Reduce the pitch error, tooth profile error, runout error and lead error.
- Grind teeth to improve the accuracy as well as the surface finish.

(2) Use Better Surface Finish on Gears

- Grinding, lapping and honing the tooth surface, or running in gears in oil for a period of time can also improve the smoothness of tooth surface and reduce the noise.

(3) Ensure a Correct Tooth Contact

- Crowning and end relief can prevent edge contact.
- Proper tooth profile modification is also effective.
- Eliminate impact on tooth surface.

(4) Have A Proper Amount of Backlash

- A smaller backlash will help reduce pulsating transmission.
- A bigger backlash, in general, causes less problems.

(5) Increase the Contact Ratio

- Bigger contact ratio lowers the noise. Decreasing pressure angle and/or increasing tooth depth can produce a larger contact ratio.
- Enlarging overlap ratio will reduce the noise. Because of this relationship, a helical gear is quieter than the spur gear and a spiral bevel gear is quieter than the straight bevel gear.

(6) Use Small Gears

- Adopt smaller module gears and smaller tip diameter gears.

(7) Use High-Rigidity Gears

- Increasing face width can give a higher rigidity that will help in reducing noise.
- Reinforce housing and shafts to increase rigidity.

(8) Use High-Vibration-Damping Material

- Plastic gears will be quiet in light load, low speed operation. Care should be taken, however, to the reduced ability to operate at elevated temperatures.
- Cast iron gears have lower noise than steel gears.

(9) Apply Suitable Lubrication

- Lubricate gears sufficiently.
- High-viscosity lubricant will have the tendency to reduce the noise.

(10) Lower Load and Speed

- Lowering rotational speed and load as far as possible will reduce gear noise.

11 A METHOD FOR DETERMINING THE SPECIFICATIONS OF A SPUR GEAR

Illustrated below are procedural steps to determine specifications of a spur gear.

Steps

① Count up how many teeth a sample spur gear has. $z =$ ☐

② Measure its tip diameter. $d_a =$ ☐

③ Estimate a rough approxmation of its module assuming that it has unshifted standard full depth tooth, using the equation:

$$m = \frac{d_a}{z+2} \qquad m \fallingdotseq \square$$

④ Measure the span measurement of k, span number of teeth. Also, measure the same of $k-1$. Then calculate the difference.

Span number of teeth $k =$ ☐ Span measurement $W_k =$ ☐

" $k-1 =$ ☐ $W_{k-1} =$ ☐

The difference = ☐

⑤ This difference represents $p_b = \pi m \cos \alpha$. Select module m and pressure angle α from the table on the right.

$m =$ ☐

$\alpha =$ ☐

⑥ Calculate the profile shift coefficient x based on the above m and pressure angle α and span measurement W.

$x =$ ☐

Table : Base pitch p_b

Module	Pressure angle 20°	Pressure angle 14.5°	Module	Pressure angle 20°	Pressure angle 14.5°
1	2.952	3.042	8	23.619	24.332
1.25	3.690	3.802	9	26.569	27.373
1.5	4.428	4.562	10	29.521	30.415
2	5.904	6.083	11	32.473	33.456
2.5	7.380	7.604	12	35.425	36.498
3	8.856	9.125	14	41.329	42.581
3.5	10.332	10.645	16	47.234	48.664
4	11.808	12.166	18	53.138	54.747
5	14.760	15.208	20	59.042	60.830
6	17.712	18.249	22	64.946	66.913
7	20.664	21.291	25	73.802	76.037

NOTE: This table deals with pressure angle 20° and 14.5° only. There may be the case where the degree of pressure angle is different.

12 A METHOD FOR DETERMINING THE SPECIFICATIONS OF A HELICAL GEAR

Helix angle is what differs helical gear from spur gear. And it is necessary that helix angle is measured accurately.
Gear measuring machine can serve for this purpose. When the machine is unavailable you can use a protractor to obtain a rough figure.
Lead p_z of a helical gear can be presented with the equation:

$$p_z = \frac{\pi z m_n}{\sin\beta}$$

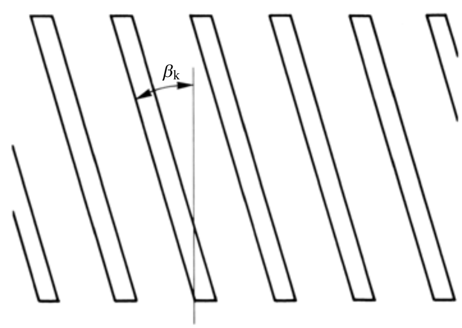
Given the lead p_z, number of teeth z normal module m_n, the helix angle β can be found with the equation:

$$\beta = \sin^{-1}\left(\frac{\pi z m_n}{p_z}\right)$$

The number of teeth z and normal module m_n can be obtained using the method explained in 11 above. In order to obtain p_z, determine d_a by measuring tip diameter. Then prepare a piece of paper. Put ink on the outside edge of a helical gear, roll it on the paper pressing it tightly. With the protractor measure angle of the mark printed on the paper, β_k.

Lead p_z can be obtained with the following equation.

$$p_z = \frac{\pi d_a}{\tan\beta_k}$$



Measuring helix angle on tooth tips